One Factor Models for the ABS Correlation Market Pricing TABX Tranches

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Abstract

In this paper we look at one factor models for TABX, the tranches of ABX.HE. Both the Gaussian copula and the Lévy base correlation method are applied to price the tranches. We describe adaptations made to the standard recursive approach for pricing TABX. Next we compare the gaussian copula formulation with the Lévy base correlation method. We show that ABX.HE and TABX reveal important information when compared to the traditional subordination levels used by the rating agencies before the credit crunch. Finally we present some results obtained with our pricing methodology and we finish by showing the necessity for market participants to be more transparent on the prepayment assumptions.

1 Introduction

In recent years there has been an enormous growth in the credit derivatives market from hundreds of billions in 1997 to more than 8 trillion USD in 2005, as can be seen in publications by the British Bankers Association. Moreover what began as a source for arbitraging regulatory capital using single name credit default swaps (CDS) has rapidly evolved to a full blown market of synthetic standardised indices.

Standardised synthetic Collateralized Debt Obligation (CDO) indices are supposed to bring liquidity to the credit market permitting one to take long and short positions for both investment and hedging purposes. Additionally the indices contain essential information for the short term management of the credit exposure and economic capital of a financial conglomerate. We refer to Garcia et al. [GGL08] for details.

First we have seen the development of the indices for corporates (iTraxx/CDX) and more recently the indices for subprime home equity mortgage backed securities (ABX.HE), commercial

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mortgage backed securities (CMBX) and recently (leveraged) loans (LevX/LCDX). An evidence of the importance of the MBS market and consequently the ABX and TABX instruments, for the whole economy is that recently, in 2006 and 2007, the issuance of long term debt has been almost twice the size of the corporate debt. Another indication of the importance of the standardised index ABX.HE has been the launch of TABX, the tranches for ABX.HE. Empirical evidence indicates this index to have been essential in the hedging on the credit crunch that began in mid 2007.

The methodology widely used by market participants in the pricing of standardised corporate indices and their tranches (iTraxx/CDX) is based on the one-factor gaussian copula and is called the *recursive approach*. The purpose of this paper is to adapt the recursive algorithm to price the TABX tranches by using both the gaussian copula and the Lévy base correlation method. When pricing TABX we are confronted with three sort of difficulties.

First, the instruments underlying the contract are CDS's on ABS and contrary to the corporate CDS case, their prices are not easily available. Second, a more fundamental issue is that the notional underlying the contract is amortising. The amortization schedule depends on the assumptions of *preprayment* on the mortgage pool. One possible estimation of the prepayment is available on the remittance report of the underlying ABS. Third, market participants.have not (yet) agreed on a standard algorithm to price these indices.

In this paper we propose an adaptation to the standardised recursive approach that takes into account the amortization schedule to price TABX. We will assume the prepayments are as given in the remittance reports. We show the results using the standard gaussian copula and the Lévy base correlation method.

The remainder of this paper is organised as follows. In Section 2 we review the generic one factor model for valuation of CDO tranches and define Lévy base correlation. Valuation formulas for both the ABS bond and the CDS on it are given in Section 3. In Section 4 we describe a simplified version of the model in order to explain the sensitivity to the input parameters. In Section 5 we show the results using the simplifications described in Section 4. Results related to prepayment assumptions and model calibration are given in Section 6. In Section 7 the implications of this pricing model are discussed. Finally our conclusions are presented in Section 8.

2 Generic One Factor Model

The one factor Gaussian copula model using the so called *recursion algorithm* was first introduced by Andersen et al. [ASB03] and is in widespread use by market participants. In what follows we give a brief description of the generic one factor algorithm.

Consider a portfolio of N names and fix a time horizon T. It is standard market practice to assume the default process to follow an inhomogeneous Poisson process and as such for any $0 \leq t \leq T$ the default times τ_i and default intensities $\lambda_i(t)$, $i = 1, \ldots, N$, satisfy

$$\mathbb{P}(\tau_i > t) = \exp\left(-\int_0^t \lambda_i(u)\mathrm{d}u\right) \tag{1}$$

where $\mathbb{P}$ is the risk-neutral probability measure. In a 1-factor model of portfolio defaults, a single systemic factor X is introduced, conditional upon which all default probabilities are independent. The single name survival probabilities $\mathbb{P}(\tau_i > t)$ are typically implied from the credit default swap (CDS) market. The fair spread of a CDS balances the present value (PV) of the contingent leg, that is the present value of losses in case of defaults, and the present value of the fee leg.

The key step in valuing CDO tranches is to compute the joint loss distribution. In the recursion algorithm one computes a discretised version of the conditional loss distribution by means of a simple recursion formula. The unconditional loss distribution is found by integrating over the market factor. Analogous to the CDS case the fair spread of a CDO tranche balances the present value of the fee leg and the present value of the contingent leg. In the base correlation framework, the expected loss on a tranche [A – D] is computed as the difference of the expected loss of two equity tranches [0 – D] and [0 – A]:

$$\mathbb{E}L[\text{A–D}] = \mathbb{E}L[\text{0–D}; \rho_D] - \mathbb{E}L[\text{0–A}; \rho_A]. \tag{2}$$

In *latent variable* models default occurs when a certain variable A_i falls below a threshold K_i which is implied from CDS prices. The so called *market* or *systemic* factor X and the *idiosyncratic* factor $X^{(i)}$ are random variables whose functional form depends on model assumptions. In the generic one factor Lévy model the latent variable is represented as

$$A_i = X_\rho + X^{(i)}_{1-\rho}, \quad i = 1, \ldots, N, \tag{3}$$

where X and $X^{(i)}$ are independent and identically distributed variates and each A_i has the same (infinitely divisible) distribution function H_1. Note that for $i \neq j$, we have $\text{Corr}[A_i, A_j] = \rho$. The threshold implied from the CDS risk neutral probability of defaults is given by

$$K_i(t) = H_1^{[-1]}(p_i(t)). \tag{4}$$

The conditional default probability of firm i given the value y for the systemic factor is given by

$$p_i(y; t) = H_{1-\rho}(K_i(t) - y). \tag{5}$$

Several authors have described one factor models using distributions other than the standard normal distribution, see e.g. Moosbrucker [Moo06], Kalemanova et al. [KSW07], Albrecher et al. [ALS06], Hooda [Hoo06] and Baxter [Bax06].

We consider two choices for the distributions of the latent variables. First, note that the classical Gaussian copula model is a special case of this generic one factor model, in which the normal distribution is used. Second, we use a shifted Gamma distribution and set $X_t = \sqrt{a}t - G_t$, in which G_t follows a Gamma$(at, \sqrt{a})$ distribution so that $\mathbb{E}[X_1] = 0$ and $\text{Var}[X_1] = 1$. Both the cumulative distribution function $H_t(x; a)$ of X_t, and its inverse $H_t^{[-1]}(y; a)$, can easily be obtained from the Gamma cumulative distribution function and its inverse.

Lévy base correlation is defined as the base correlation in the shifted Gamma model with fixed $a = 1$. For more details on Lévy base correlation we refer to Garcia et al. [GGMS07]. A comparison with the Gaussian base correlation can be found in Garcia and Goossens [GG07b]. Arbitrage-free interpolations techniques for base correlation methods based on base expected loss have been described by Garcia and Goossens [GG07a].

3 Amortizing Bond and CDS

In this section we give valuation formulas for an ABS bond and its CDS. As mentioned earlier the main difference with respect to corporate case is that, due to amortization and prepayment, the notionals are time-dependent. Assume the amortized (ABS) bond is a floater paying a coupon C above Libor. Its value B is computed as

$$\begin{aligned} B &= \sum_{i=1}^{n} N_i(L_i + C)d(t_i)P_S(t_i)\Delta t_i + P_S(t_n)d(t_n)N_n \\ &+ \sum_{i=1}^{n-1} \left(N_i - N_{i+1}\right) d(t_i)P_S(t_i) + R\sum_{i=1}^{n} N_i d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right). \end{aligned} \tag{6}$$

In these equations the summations run over the payment dates t_i. We denote by L_i, N_i, $P_S(t_i)$ and $d(t_i)$ the Libor rate, the notional, the survival probability and the risk free discount factor respectively at time t_i. The recovery rate is R, and $\Delta t_i = t_i - t_{i-1}$ is the year fraction.

The value of the amortized CDS, paying a spread S on notionals N_i is computed as

$$C = (1 - R)\sum_{i=1}^{n} N_i d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right) - S\sum_{i=1}^{n} N_i d(t_i)P_S(t_i)\Delta t_i. \tag{7}$$

The fair spread of a CDS balances the present value of the contingent leg, that is the present value of losses in case of defaults, and the present value of the fee leg.

4 A Simple Model for Amortization and Prepayment

In this section we look at a simple model for amortization and prepayment and we study the impact on survival probability, duration and expected loss for a fixed price of the amortizing bond.

We make the following assumptions. A simple daycounter is used so that the payment times $t_i = i\Delta t$, are integer multiples of the constant year fraction Δt. The notionals at the payment times are given by

$$N_i = N(t_i) = \exp(-\lambda_{\text{Not}} i\Delta t) = n^i, \tag{8}$$

where λ_{Not} is the constant amortization factor, modelling *both the scheduled amortization as well as prepayments*. The discount factors are assumed to be given by

$$d(t_i) = \exp(-ri\Delta t) = d^i, \tag{9}$$

corresponding to a flat interest rate r. The default intensity λ_{Surv} is constant, so that the survival probability is given by

$$q(t_i) = \exp(-\lambda_{\text{Surv}} i\Delta t) = q^i. \tag{10}$$

We define the constant α as the product of these 3 factors

$$\alpha = dnq. \tag{11}$$

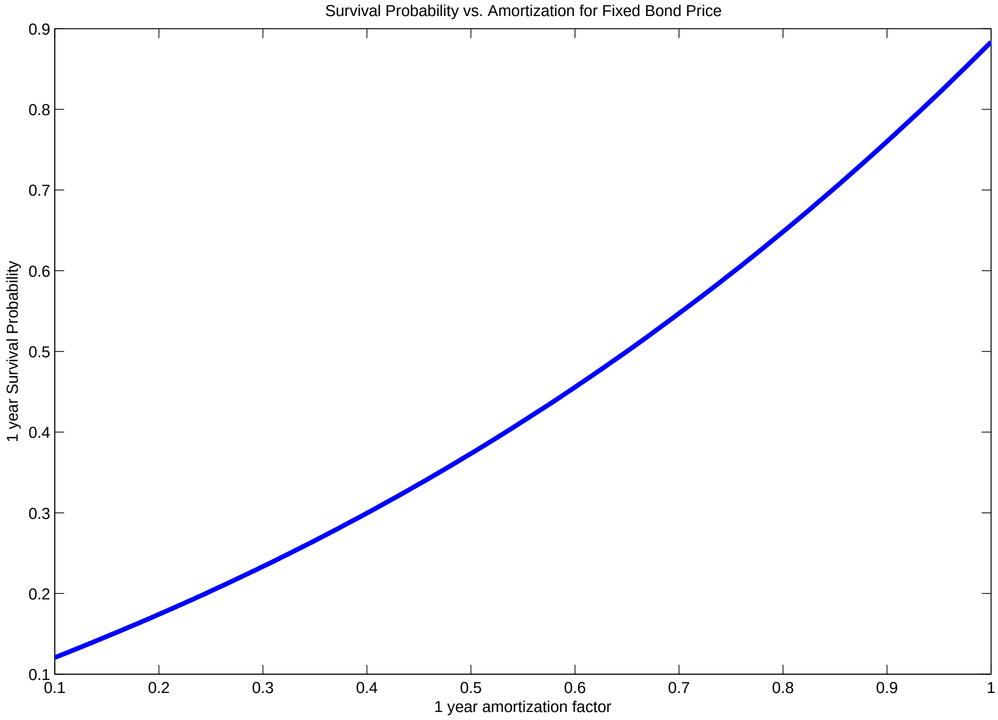


Figure 1: Survival Probability versus Prepayment for a fixed bond price.

In this paper we assume that the recovery rate $R = 0$ is zero. The price of the bond is

$$B = ((r + C)\Delta t + (1 - n))\frac{\alpha}{1 - \alpha}. \tag{12}$$

Fig. 1 shows the survival probability versus the amortization after one year for a fixed bond price. The expected loss versus the amortization is shown in Fig. 2. Finally Fig. 3 shows the Macaulay Duration versus the amortization. The following parameters were used to generate the figures. The bond price was $B = 40\%$, the coupon was $C = 2\%$ above the interest rate $r = 5\%$ and the year fraction was taken to be $\Delta t = 1/12$, corresponding to monthly payments. We conclude that for a given *fixed* bond price, if amortization increases, the duration goes down, the expected loss goes down and the survival probability goes down.

Prepayment speeds are generally available for ABS instruments. An open question is whether the prepayment assumption available in remittance reports match those embedded in the prices of synthetic traded instruments. The rest of this paper is devoted to this question.

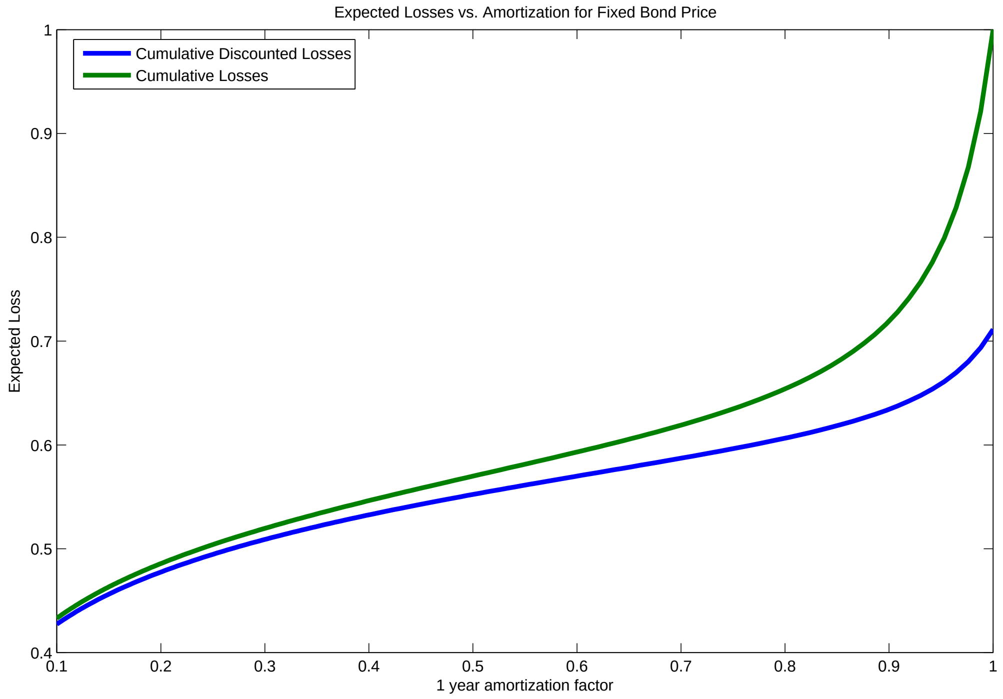


Figure 2: Expected Loss versus Prepayment for a fixed bond price.

5 ABX.HE Credit Index: Benchmark for CDS on ABS

The ABX.HE is the index for subprime home equity mortgages. A new series is supposed to be issued every 6 months. However Markit has delayed the launch of the ABX.HE 08-01 series due to limited availability of candidate underlyings as a result of the credit crunch related to subprime home equity. Five ABX.HE indices have been created, corresponding to the ratings: AAA, AA, A, BBB and BBB-. The portfolio is static and consists of 20 CDS's on subprime MBS. Hence it is unfunded. TABX is the instrument for the tranches of ABX.HE, and it has only been defined for the BBB and BBB- rated indices, respectively TABX.HE.BBB and TABX.HE.BBB-. The collateral pool for the TABX series is composed of 2 succesive ABX.HE pools, that is 40 CDS's on ABS. An important difference between ABX and TABX is that ABX covers interest shortfall, while TABX does not. Fig. 4 shows an illustration of ABX and TABX. The naming convention is based on a reference to the vintage. ABX.HE 06-2 is the index referencing MBS's issued in the second semester of 2006 and TABX-HE 07-1 06-2 BBB- refers to ABX.HE.BBB- 07-1 and ABX.HE.BBB- 06-2. Fig. 5 shows historical prices of the ABX.HE.BBB- indices for the series 06-1, 06-2, 07-1 and 07-2. Historical prices of tranches of TABX.HE.BBB- 06-2 07-1 are shown in Fig. 6, while Fig. 7 shows historical prices of tranches of TABX.HE.BBB- 07-1 07-2.

When pricing TABX we have run into several difficulties. The pricing algorithm needs quotes of CDS on ABS as an input for the evaluation of default probabilities. A first difficulty is that

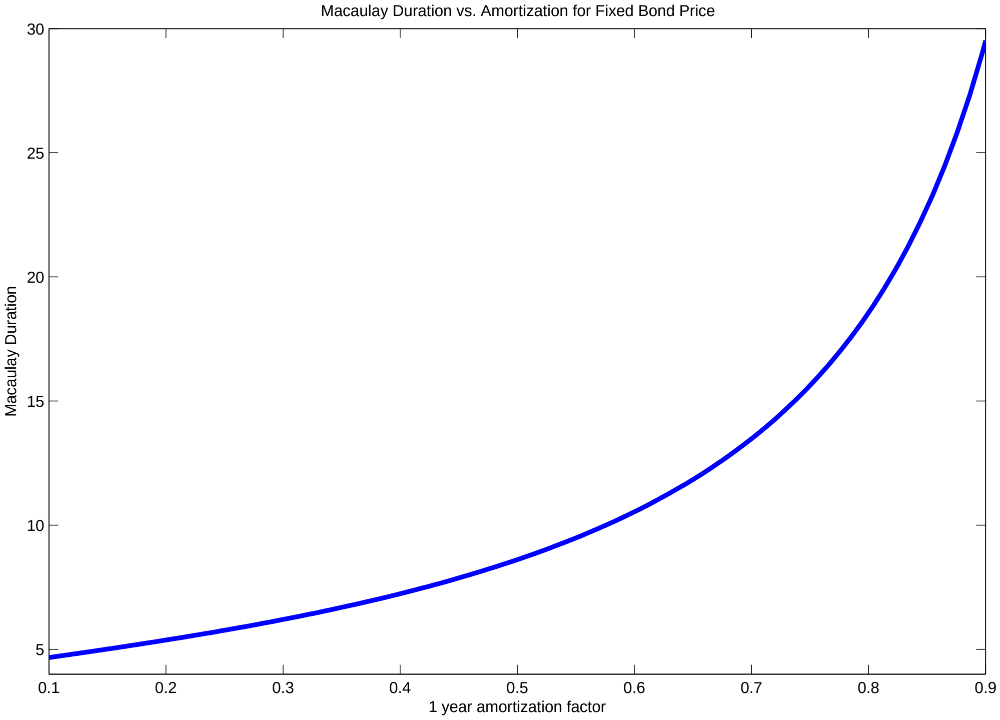


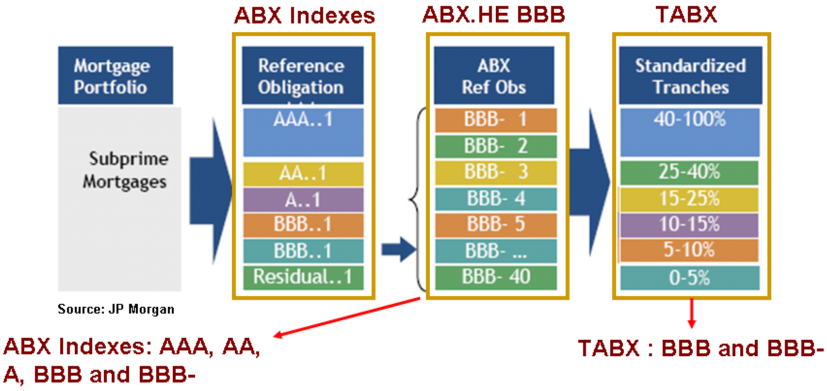
Figure 3: Macaulay Duration versus Prepayment for a fixed bond price.

those quotes are not readily available. When quotes are available, it is only a single quote, corresponding to the legal maturity. Hence we do not have a term structure of quotes for different maturities. For this reason we have used bond prices of the underlying ABS's to imply default probabilities.

A second difficulty is that there is *no market accepted standard approach* to determine the prepayment for pricing purposes. The prepayment assumptions are *key* to the pricing algorithm as will become clear. In this case we took prepayment assumptions from the remittance reports. Additionaly we varied the prepayment assumptions to cope with much more stressed scenarios that may be priced in by market participants.

A third difficulty is related to the order of the calibration process. Both a bottom up approach, starting from the equity tranche and moving up the capital structure, or a top down approach, starting from the super senior and moving down the capital structure, can be applied to calibrate to the observed prices. At the time of pricing the senior tranches of TABX were the most liquid ones. Hence these quotes are more reliable, making the top down approach seem the preferred choice. For the corporate indices (iTraxx and CDX) however the standard approach is bottom up. We have applied both bottom up and top down calibration.

In the next section we give some results obtained with our pricing algorithm. We also discuss the implications for the securitization business model for banks and for the whole financial



Source: JP Morgan

Figure 4: ABX.HE credit indices and its tranches.

	06-1	06-2	07-1	07-2	CDS	Cash
AAA	53	103	116	164	n/a	48
AA	216	473	658	613	n/a	225
A	649	1323	1733	1655	700	800
BBB	1800	2490	2547	2406	900	900
BBB-	2510	3062	3007	2508	1200	1200

Table 1: Spreads (in bp) versus ratings for different instruments (Source: Markit).

system in general.

6 Prepayment and Model Calibration

In this section we present results using both the Gaussian copula and the Lévy base correlation method for TABX 06-02 07-01. The market data is taken on August 10, 2007. We have assumed that the prepayment speeds for the bond and the CDS on it are equal

$$\mathrm{CPR}_{\mathrm{CDS}} = \mathrm{CPR}_{\mathrm{Bond}} \tag{13}$$

and that the same holds for the default intensities

$$\lambda_{\mathrm{CDS}} = \lambda_{\mathrm{Bond}}. \tag{14}$$

Fig. 8 shows the results of the base correlation for the bottom up approach. In this case the [0%–5%] tranche price determines $\rho(5\%)$, the base correlation parameter at the 5% attachment point, the [5%–10%] tranche price determines $\rho(10\%)$ and so on. Observe that the base correlation curves for both methods have the same shape. Both are decreasing with increasing

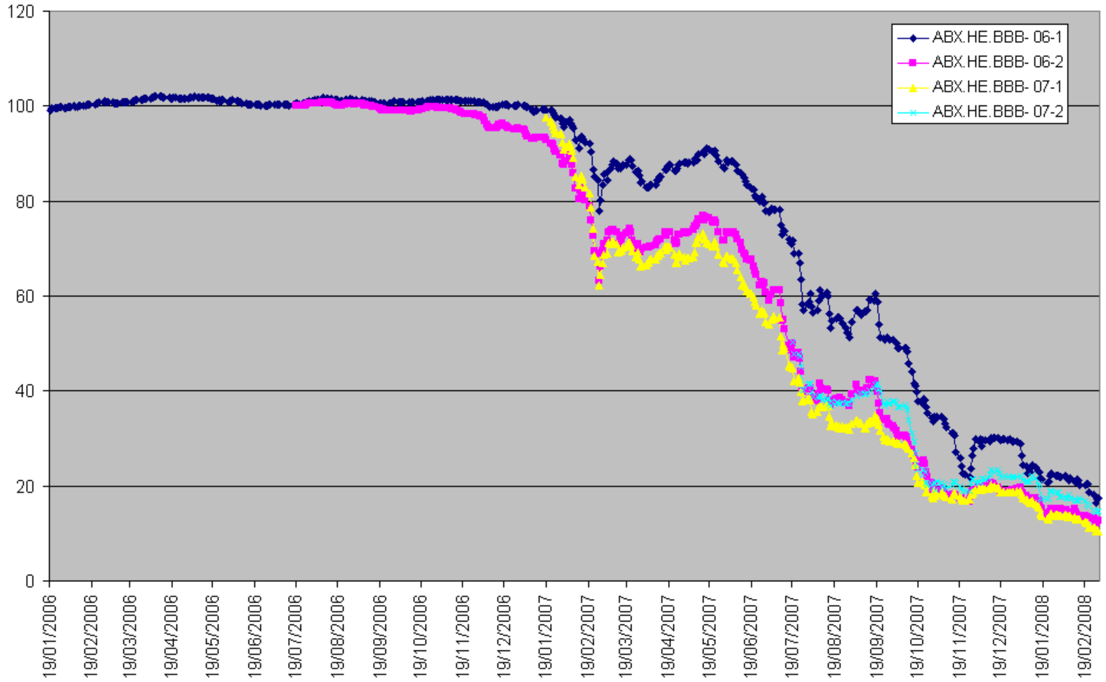


Figure 5: ABX.HE.BBB- prices, series 06-1, 06-2, 07-1 and 07-2

attachment points, corresponding to tranches with equity behaviour. This is the equity tranche part of the base correlation smile. Also note that the Lévy base correlation values are larger than the corresponding Gaussian base correlation values, which is consistent with our findings in our earlier paper [GG07a]. In the bottom up approach the [25%–40%] tranche price determines $\rho(40\%)$ and we are left with a big gap between the implied price and the observed price for the [40%–100%] tranche. It turns out that both methods fail to find a value for $\rho(40\%)$ which correctly prices the [40%-100%] tranche, given the assumed prepayment speeds and default intensities. We concluded that indeed the prepayment assumptions in the remittance reports do not correspond to the market expectations for prepayments as represented in the super senior part of TABX.

For the top down calibration, we have changed our assumptions. The prepayment speeds and default intensities were modified as follows. The prepayment speeds were assumed to be 4 times slower:

$$\text{CPR}_{\text{CDS}} = 0.25\text{CPR}_{\text{Bond}} \tag{15}$$

and the default intensities have been doubled:

$$\lambda_{\text{CDS}} = 2\lambda_{\text{Bond}}. \tag{16}$$

We have done this believing that indeed that the synthetic instruments were already pricing lower

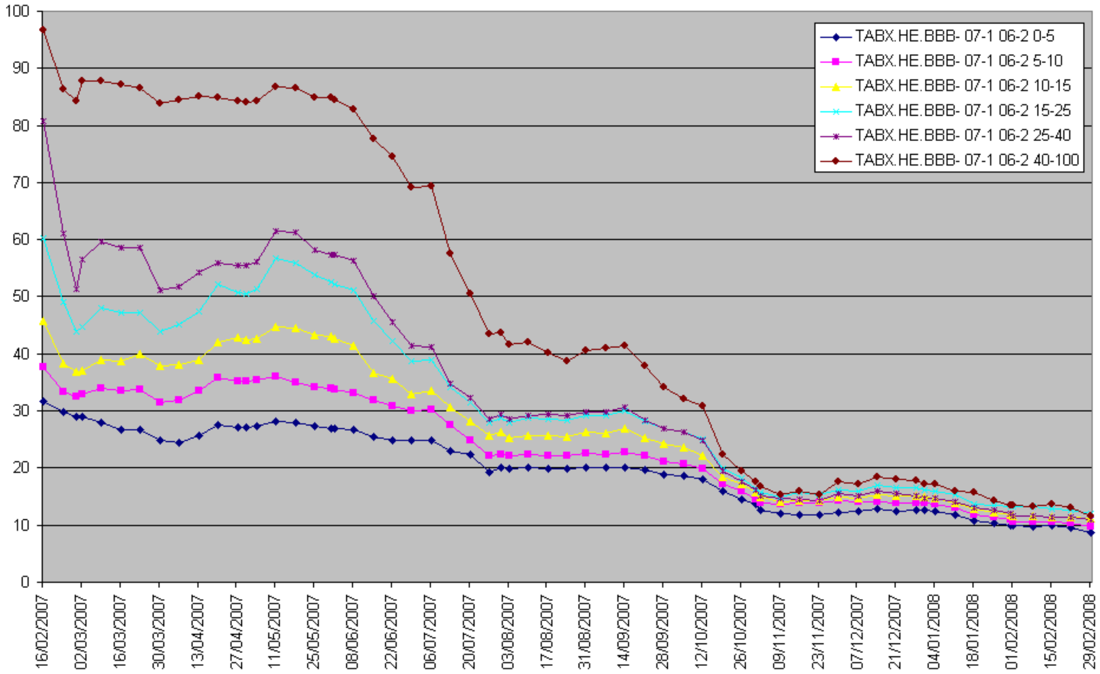


Figure 6: TABX.HE.BBB- 06-2 07-1 prices of tranches

prepayment speeds than reported in the remittance reports, see IFR Magazine Oct 13, 2007 (p41): “Lower Prepayments are coming”!

Fig. 9 shows the base correlation curves for a top down calibration. Observe that the base correlation curves for both methods have the same shape. Both are increasing with increasing attachment points corresponding to senior tranches like behaviour. This is the senior tranches part of the base correlation smile. Also note that the Lévy base correlation values are smaller than the corresponding Gaussian base correlation values, which is consistent with our findings in our earlier paper. Starting from the most liquid tranches one cannot find base correlation values that match the prices of the equity tranches.

We can conclude that with a bottom up calibration a big gap between the implied price and the observed price for the most liquid super senior tranche is seen. Calibrating to the observed price for the super senior tranche is not possible with the initial assumptions, based on remittance reports and bond prices. If we assume lower prepayment speeds and higher default intensities, senior tranches can be matched. However a top down with these modified assumptions cannot match the equity and junior mezzanine tranches.

In the next section we discuss the implications of these calibration issues in more detail. We make a case for the necessity of transparency to support the securitization business model used by large financial institutions in the last couple of years.

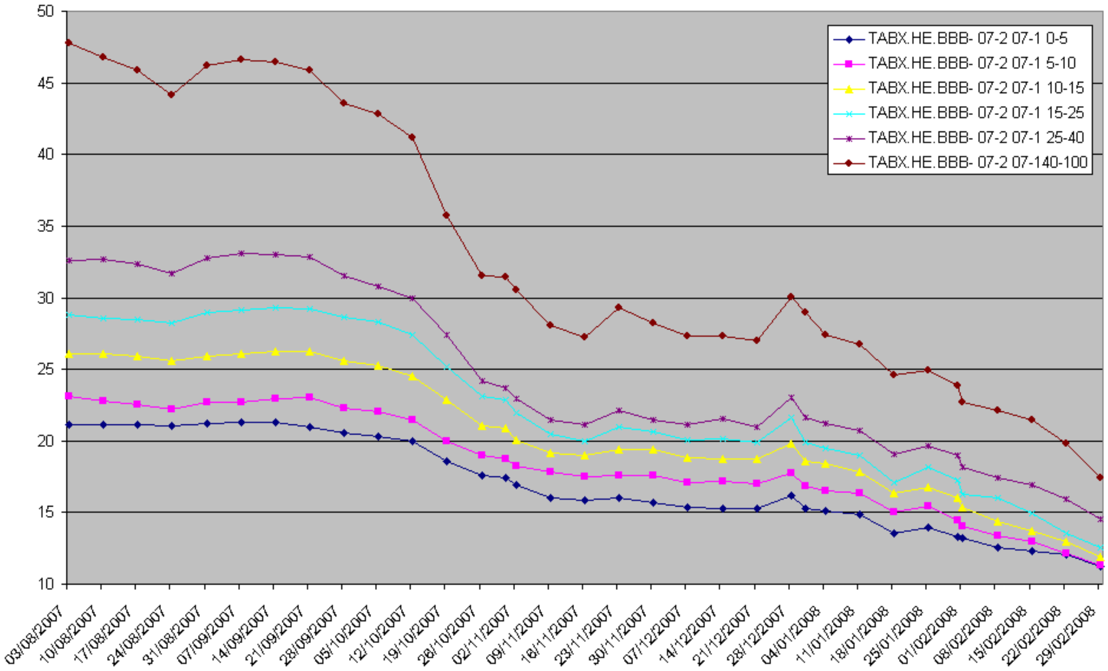


Figure 7: TABX.HE.BBB- 07-1 07-2 prices of tranches

7 Pricing Model Implications

There are two important points related to TABX tranche pricing. First, the absence of reliable CDS data. Second, the prepayment speeds in the remittance reports are too high to generate the amount of losses implied in the tranch quotes.

Note that we are already assuming the recovery rate to be 0. Looking at the corporate indices, we have only recently observed prices on the senior tranches that are too high to be handled by the standard gaussian copula algorithm. Pricing the CDX [15%–30%] senior tranche with a base correlation value equal to 100% currenty results in a price gap larger than 10%. A possible solution, nowadays being proposed by market participants, is to reduce the recovery rate from 40% to 30% or even 25%. In our TABX pricing here we are already past this point, as the recovery rate is assumed to be 0.

However given the importance of the MBS market and the securitization activity, we think the market needs to come up with a simplified and uniform assumption for the prepayment, let us call it the *risk neutral prepayment assumption*, for pricing purposes that would bring more transparency to TABX tranches. In what follows we discuss this in more detail.

The recent creation of the standardised credit indices has brought an enormous amount of *technological innovation* to the credit market. Those instruments have helped to increase liquidity in at least two ways. First, on a deal by deal basis they have helped advancing the *price*

07-1 spread	Aug 10 price	Oct 26 price	tranche	07-2 spread	Oct 26 price
500	20.05	14.46	0-5	500	17.57
500	22.36	15.85	5-10	500	18.96
500	25.77	17.19	10-15	500	21.09
500	28.77	18.17	15-25	500	23.10
267	29.11	17.67	25-40	500	24.20
72	41.99	19.38	40-100	410	31.50

Table 2: Prices of TABX-HE 07-1 06-2 BBB- and TABX-HE 07-2 07-1 BBB- tranches (Source: Markit).

ABS rating	thickness	support	CDO rating	thickness	support
AAA	80%	20%	AAA	80%	20%
AA	5%	15%	AA	10%	10%
A	6%	9%			
BBB+	2%	7%			
BBB	1%	6%	BBB	5%	5%
BBB-	1%	5%			
BB	1%	4%			
Equity	4%	0%	Equity	5%	0%

Table 3: Typical Rating of Subprime ABS and of a CDO Capital Structure

discovery process of more illiquid bespoke deals. By increasing the transparency in the prices for the whole capital structure, the indices help market particpants to have a view on prices for the capital structure of bespoke deals. Second, these liquid indices may be used as a portfolio management tool to completely or partly hedge a credit portfolio. This explains the strong growth those instruments experienced in the period from 2002 untill 2007. Observe that in order to hedge or price portions of the capital structure of a credit corporate portfolio one uses the standardised corporate credit indices and their tranches, for which a well known and understood standard methodology is available. During the credit crunch there has been no major problem with CDO tranches of corporates whose indices are well understood and transparent.

A simple evidence of the mispricing of risk in the CDO of ABS asset class is the following. Table 3 shows the typical support levels and thicknesses to attain a certain rating for both an ABS and a CDO. For the CDO the typical collateral comprises about 100 ABS notes mostly rated BBB and BBB- and about 5% to 10% rated BB. In order to get the AAA rating the tranch would need a subordination level of 20%. Fig. 6 shows the historical prices of the tranches of TABX.HE.BBB- 06-2 07-1. Observe that the tranche [40%-100%], which has a subordination level of 40%, has been trading at levels below 90% already since March 2007 . This means that

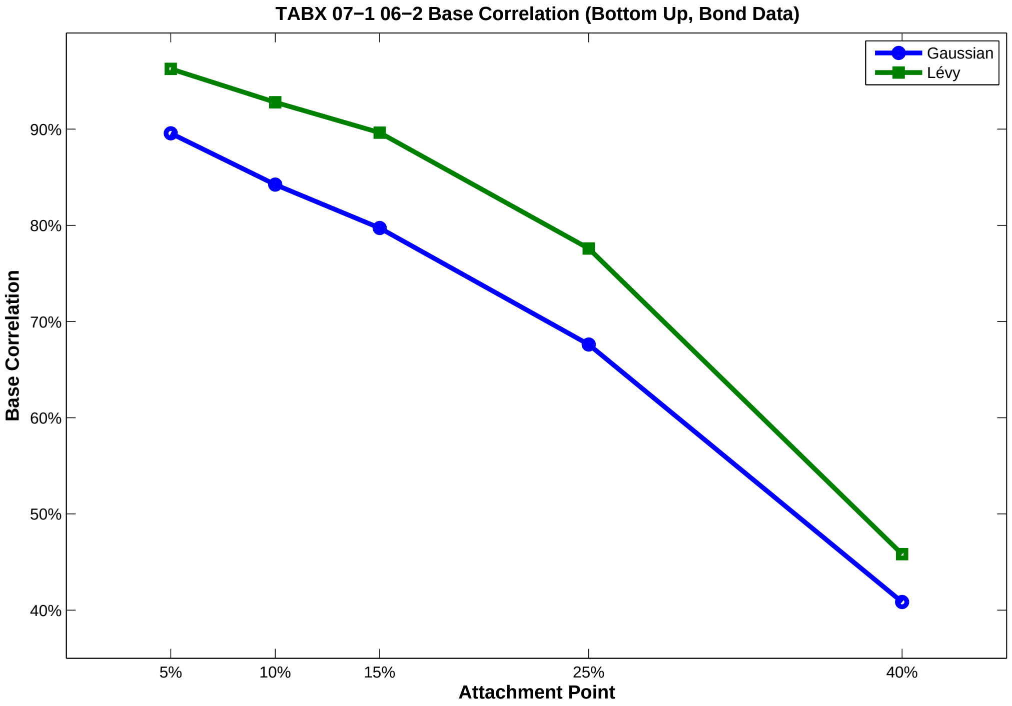


Figure 8: Gaussian and Lévy base correlation curves for the *bottom up* calibration, assuming prepayment speeds as given in remittance reports and default intensities implied from bond prices.

although from a rating perspective as seen in Table 3 TABX.HE [40%-100%] would be rated AAA, the prices were implying a far lower rating. This is the core problem underlying the credit turmoil that began in 2007.

As reported in Garcia et al. [GGL08] one of the main reasons behind the credit crunch has been the misunderstanding of the risk of CDO of ABS's. For the case of a portfolio of ABS's however there is neither a standard model nor data available to price the standardised credit indices and their tranches. The importance of this point cannot be underestimated. The size of the mortgage market is about 6–8 trillion USD. Moreover the securitization activity has been a key driver for the spectacular increase in efficiency on the use of capital by large financial institutions. The problem with the CDO of ABS's is that the indices have not been transparent enough to offer the same insights as those offered by the corporate indices. In our opinion, transparency in terms of models, data and a standard risk neutral prepayment assumption for pricing purposes, are essential ingredients in the recipe for revitalizing this market.

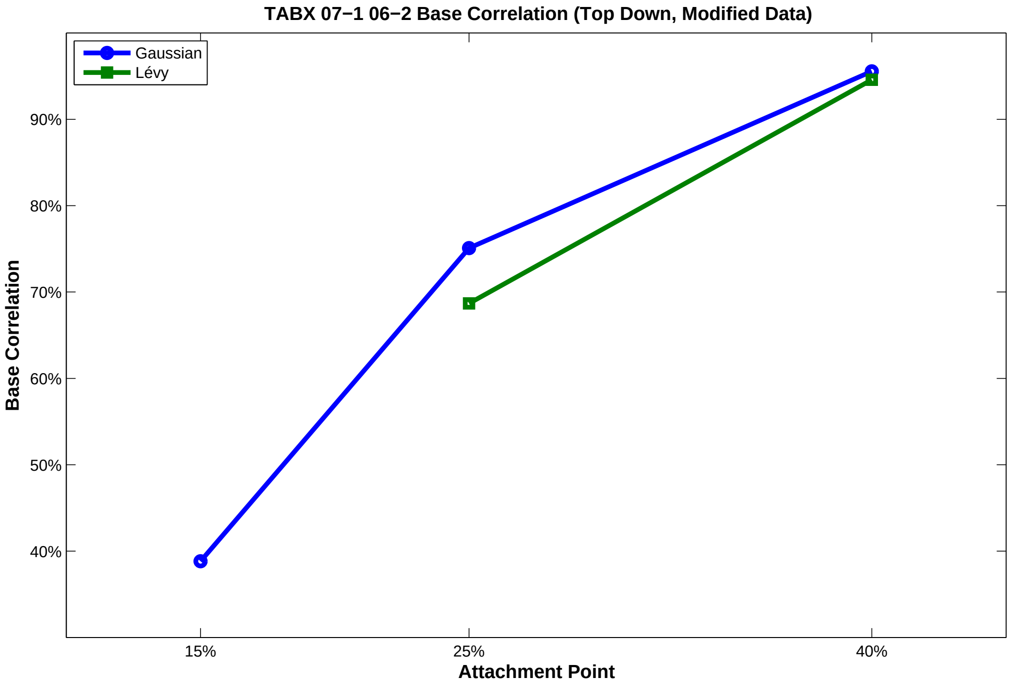


Figure 9: Gaussian and Lévy base correlation curves for the *top down* calibration, assuming lower prepayment speeds and higer default intensities.

8 Conclusions

In this paper we have outlined one factor models for TABX tranche pricing. We have shown how to adapt the standard market approach to price TABX using both the Gaussian copula and the Lévy base correlation method. Several hurdles need to be taken before TABX can be priced transparently and hence used efficiently. First, there is only limited data available for the collateral, the CDS is particularly troublesome. Second, an assumption on prepayment is essential for pricing purposes. Using the values available in the remittance report of the underlying ABS, one will not be able to recover observed market prices for the most liquid super senior tranche. Given the size and importance of the securitization activity in general and of the MBS market in particular we believe that market practioners need to come up with much more transparency in dealing with the standardised credit index and its tranches. Those indices are the natural instruments not only essential for pricing purposes but also for managing a credit portfolio.

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