Lévy Base Correlation Mapping

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Abstract

In an earlier paper we treated the concept of Base Expected Loss (BEL) (both for the Gaussian Copula and Lévy Base Correlation models) as an arbitrage free approach to interpolate the base correlation curves for pricing non standard tranches of the standardised credit indices. In this paper we extend the approach further by using the technique to price CDO's of non-standardised portfolios (so called *bespoke* CDO's). That is the framework is developed in the context of *correlation mapping*. Additionally we compare the correlation mapping methodology developed here with alternative approaches largelly used with practicioners.

1 Introduction

Since the introduction of the one factor Gaussian copula model for pricing synthetic Collateralized Debt Obligation (CDO) tranches by Andersen et al. [ASB03] correlation is seen as an exogenous parameter used to match observed market quotes. First the market adopted the concept of *implied compound correlation*. One of the problems of this approach resides in its unsuitability for interpolation when pricing non-standard tranches. The current widespread market approach is to use the concept of *base correlation* (BC) introduced by McGinty et al. [MBAM04]. In the base correlation methodology only equity or base tranches (with attachment point 0) are considered. The price of a tranche [A - D] is calculated using the two equity tranches with A and D as detachment points. The BC concept is supposed to be suitable for interpolation both for non-standardised tranches as for bespoke portfolios.

The methodology however has several weaknesses: it is very sensitive to the interpolation technique used, it may not be arbitrage-free and finally it does not provide any guidance on how to extrapolate the curve, especially below the 3% attachment point[1].

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[1]We show in this paper this last requirement becomes particularly important when pricing tranches of bespoke CDO's

A recent solution to the difficulties we mentioned above is to use expected loss. Garcia and Goossens [GG08] looked at the concept of base expected loss (both in the Gaussian copula and Lévy based framework) for pricing non-standardised tranches of standard credit indices. The approach brought two important contributions, first the use of base expected loss for arbitrage-free pricing and second the use of the base correlation methodology in non-Gaussian one factor models.

As described in Garcia et al. [GGL08] two important uses for the standardised credit indices are to bring liquidity (to the underlying class of instruments) and to be used for dynamic portfolio management of a credit portfolio. Annedoctical evidence (see e.g. Tett and Larsen [TL08]) during the credit crunch that began in Jun 2007 shows that indeed the indices are the ideal instruments not only for hedging but also for the mark to market of bespoke credit portfolios. In this paper we extend the base expected loss framework (described in Garcia and Goossens [GG08]) to the pricing of tranches of non-standard (or bespoke) portfolios. Similar to the management of equity portfolios where one needs to determine the appropriate volatility parameter in the case of a credit portfolio one needs to determine the appropriate *correlation* for the bespoke portfolio. The technique of projecting the capital structure of a bespoke portfolio on a credit index for pricing purposes (under the use of 1-factor models) is called *correlation mapping*. Correlation mapping (CM) techniques are common knowledge among praticioners involved in pricing of bespoke CDO's. This paper complements the known techniques in three rather important ways. First we show results of CM techniques using arbitrage free interpolation approach based on expected loss (which is independent of the model framework used). Second we apply the techniques to both the gaussian approch and Lévy based framework. Third, the tests are done using realistic data for the iTraxx and CDX indices.

Given some of the problems faced by practicioners (during the recent credit crunch) in determining prices of portfolios of structured finance deals we feel some words of caution may be appropriate before proceeding.

First, market practioners may certainly use pattern recognition techniques in the determination of tranche prices of bespoke portfolios. In general such a framework means a multidimensional regression of the unknown price of the bespoke tranche into the prices of (assumed) liquid tranches. Although those techniques might be much more appealing for an experienced market practicioner (say a trader or a credit portfolio manager with many years experience) they are not at all straightforward to be used in the determination of portfolio hedge parameters. Additionaly those techniques are based on the availability of prices of other liquid tranches that are used in the price discovery process. In this note we assume the existence of one index that is liquid and a transparent and well understood (known) pricing algorithm whose model parameters are also well known by market partcipants (pricing agents).

Second, the securitization business model used by large financial institutions has been a very important ingredient behind the enormous growth of economic activity of recent years. The recent problems triggered by risk mispricing of CDO's of ABS's means that the cost of capital of structured products in the portfolios of banks will increase, possibly causing an impact on the whole economic activity in case the securities are put on held to maturity. A possible solution for those issues while reviving the securitization activity is to have securitization instruments being put on mark to market and the portfolio being managed dynamically with standardised credit indices (see Garcia et al. [GGL08] for details). This being the case mapping techniques for bespoke tranches into credit indices become very important technologies. In this note we show results of some of those techniques.

The remainder of this paper is organised as follows. In section 2 we review the generic one factor model for valuation of CDO tranches. In section 3 we briefly define the base expected loss concept, describe its properties and outline the upper and lower bounds it must satisfy. In section 4 we define alternative correlation mapping techniques commonly used among practicioners, and additionally we briefly describe how the rating agency approach can be used for correlation mapping. Numerical results are given in section 5. Finally our conclusions are presented in section 6.

2 Generic One Factor Model

The one factor Gaussian copula model using the so called *recursion algorithm* was first introduced by Andersen et al. [ASB03] and is in widespread use by market participants. In what follows we give a brief description of the generic one factor algorithm. Consider a portfolio of N firms and fix a time horizon T. It is standard market practice to assume the default process to follow an inhomogeneous Poisson process and as such for any $0 \leq t \leq T$ the default times τ_i and default intensities $\lambda_i(t)$, $i = 1, \ldots, N$, satisfy $\mathbb{P}(\tau_i > t) = \exp\left(-\int_0^t \lambda_i(u)\mathrm{d}u\right)$ where $\mathbb{P}$ is the risk-neutral probability measure. In a one factor model of portfolio defaults, a single systemic factor X is introduced, conditional upon which all default probabilities are independent. The single name survival probabilities $\mathbb{P}(\tau_i > t)$ are typically implied from the credit default swap (CDS) market. The fair spread of a CDS balances the present value of the contingent leg C, that is the present value (PV) of losses in case of defaults, and the present value of the fee leg F.

The key step in valuing CDO tranches is to compute the joint loss distribution. In the recursion algorithm one computes a discretised version of the conditional loss distribution by means of a simple recursion formula. The unconditional loss distribution is found by integrating over the market factor. Analogous to the CDS case the fair spread of a CDO tranche balances the present value of the fee leg F and the present value of the contingent leg C. In the base correlation framework, the expected loss on a tranche is computed as the difference of the expected loss of two equity tranches. For more details on base correlation we refer to McGinty et al. [MBAM04].

In *latent variable* models default occurs when a certain variable A_i falls below a threshold K_i that is implied from CDS prices. The so called *market* or *systemic* factor X and the *idiosyncratic* factor $X^{(i)}$ are random variables whose functional form depends on model assumptions. In the generic one factor Lévy model the latent variable is represented as

$$A_i = X_\rho + X^{(i)}_{1-\rho}, \quad i = 1, \ldots, N, \tag{1}$$

where X_t and $X^{(i)}_t$ are independent and identically distributed variates and each A_i has the same (infinitely divisible) distribution function H_1. Note that for $i \neq j$, we have $\mathrm{Corr}[A_i, A_j] = \rho$. The threshold implied from the CDS risk neutral probability of defaults is given by

$$K_i(t) = H_1^{[-1]}(p_i(t)). \tag{2}$$

The conditional default probability of firm i given the value y for the systemic factor is given by

$$p_i(y;t) = H_{1-\rho}(K_i(t) - y). \tag{3}$$

Several authors have described one factor models using distributions other than the standard normal distribution. For more details we refer to our earlier paper [GGMS07] and the references therein.

We consider two choices for the distributions of the latent variables. First, note that the classical Gaussian copula model is a special case of this generic one factor model, in which the normal distribution is used. Second, we use a shifted Gamma distribution and set $X_t = \sqrt{a}t - G_t$, in which G_t follows a Gamma$(at, \sqrt{a})$ distribution so that $\mathbb{E}[X_1] = 0$ and Var$[X_1] = 1$. Both the cumulative distribution function $H_t(x; a)$ of X_t, and its inverse $H_t^{[-1]}(y; a)$, can easily be obtained from the Gamma cumulative distribution function and its inverse. The Gamma distribution was selected as it is the most tractable choice among several distributions with comparable performance.

Lévy base correlation is defined as the base correlation in the shifted Gamma model with fixed $a = 1$. For more details on Lévy base correlation and a comparison with the Gaussian base correlation we refer to our earlier papers [GG07] and [GG08].

3 Base Expected Loss and Base Correlation

As already mentioned in the last section in the base correlation framework, the expected loss on a tranche is computed as the difference of the expected loss of two equity tranches

$$\mathbb{E}L[\text{A–D}] = \mathbb{E}L[\text{0–D}; \rho_D] - \mathbb{E}L[\text{0–A}; \rho_A].$$

In order to price bespoke tranches, non standard tranches or tranches of an older series, one may have to (directly or indirectly) interpolate or extrapolate the base correlation curve of the current series.

In the case of direct interpolation, to our knowledge, the most widespread used method among practitioners is cubic spline. It has the advantage of producing continuous curves with continuous first and second derivatives and as such avoiding sudden spread jumps that may lead to possible arbitrage opportunities. Unfortunately enough even smooth base correlation curves may be arbitrageable and it is impossible to guarantee absence of arbitrage unless all tranchlet prices are checked. Moreover the extrapolation issues outside the 3%-22% region remain.

In order to avoid arbitrage opportunities practicioners have come up with the concept of base expected loss. In Garcia and Goossens [GG08] we have proposed a cubic spline interpolation with a monotonicity filter applied to it on the base expected loss producing an arbitrage-free correlation mapping technique independent of the pricing framework. We define base expected loss $l(x)$ as the expected loss of an equity tranche at maturity

$$l(x) = \mathbb{E}[\text{Loss}(0, x, \rho_x, T)]. \tag{4}$$

This quantity is readily available in the base correlation framework. In order to avoid model arbitrage the base expected loss curve should obey two important conditions. First, base expected loss $l(x)$ is a non-decreasing function of the attachment point x, as its derivative corresponds to a probability:

$$\frac{\partial}{\partial x} l(x) = \mathbb{P}[\text{Loss} \geq x] \geq 0. \tag{5}$$

Second, base expected loss $l(x)$ cannot be positively convex, as its second derivative is minus the density of the loss distribution:

$$\frac{\partial^2}{\partial x^2} l(x) = -f_{\text{Loss}}(x) \leq 0. \tag{6}$$

A considerable advantage of using the base expected loss curve for interpolation purposes is that the expected loss is a much more intuitive concept and is in fact the quantity of interest for pricing purposes. In this approach the base correlation is implied from the base expected loss curve and not the other way around.

Note that this definition is different from what Parcell and Wood [PW07] define as discounted base expected loss, which is the present value of the contingent leg. The ideas presented in here apply to both approaches.

4 Correlation Mapping for Bespoke Portfolios

The current market approaches for pricing bespoke tranches are based on correlation mapping methodologies. The concept in a nutshell consists of assuming market invariants which are used to imply from the correlation curve of the (liquid) standardised credit indices the correlation to be input in the standard model to price bespoke tranches. The approaches we will be testing are the following:

- **No Mapping**

  This is the simplest possible approach and the base correlation for a certain attachment point on the bespoke portfolio is equal to the correlation for the same attachment point in the index portfolio:

  $$\rho_{Bespoke}(A) = \rho_{Index}(A) \tag{7}$$

  It assumes that equity base correlation is invariant to the portfolio characteristics. This rather naive method assumes that correlation depends only on the *attachment point* of the capital structure and *not* on the credit quality of the underlying portfolio. Comparisons with this approach show how prices differ due to differences in the underlying portfolios.

- **Discounted Moneyness Matching**

  One assumes that the attachment point as a fraction of the portfolio expected loss is an invariant measure to determine the cost of risk. The ratio between attachment point and portfolio expected loss is called in the market as *moneyness* and the correlation for the attachment point A is given by:

$$\rho_{Bespoke}(A) = \rho_{Index}(A \cdot \frac{\mathbb{E}[\text{Loss}_{Bespoke}(0, 100\%)]}{\mathbb{E}[\text{Loss}_{Index}(0, 100\%)]}) \tag{8}$$

It is a widespread used methodology in the market. If the bespoke and the index portfolio have the same expected losses then it will give the same results as no mapping. The main advantage of this approach is simplicity. One big disadvantage is that it does not take into account the dispersion of spreads in the portfolio. As the expected loss on the whole index does not depend on correlation it is discussable why the correlation at different attachment points should be completly determined by the expected loss on the whole index.

- **Moneyness at Maturity**

  It is very similar with the approach described in (8) with the only difference that there is no impact of the discount curve.

$$\rho_{Bespoke}(A) = \rho_{Index}(A \cdot \frac{\mathbb{E}[\text{Loss}^{mat}_{Bespoke}(0, 100\%)]}{\mathbb{E}[\text{Loss}^{mat}_{Index}(0, 100\%)]}) \tag{9}$$

- **Probability Matching**

  In this case the invariant is the probability of loss of an equity tranch at maturity. One searches for the attachment point in the index such that the cumulative loss probability at maturity equals the one of the attachment point in the bespoke portfolio:

$$\mathbb{P}[\text{Loss}^{mat}_{Bespoke} \leq A_{Bespoke}] = \mathbb{P}[\text{Loss}^{mat}_{Index} \leq A_{Index}] \tag{10}$$

  and:

$$\rho_{Bespoke}(A_{Bespoke}) = \rho_{Index}(A_{Index}) \tag{11}$$

  Observe that changes in correlation do not affect the portfolio expected loss but the the form of portfolio loss distribution. One advantage is that approach takes into account the dispersion of the spread within the portfolio. The discrete nature of the portfolio means that the approach is inherently discontinuous and one needs to use interpolation techniques to smooth out the discontinuities. The mapping might fail if the quality of the bespoke portfolio is much worse than the one of the index. We think that any mapping technique that tries to map the capital structures of two very different portfolios will be prone to errors.

- **Base Spread Matching**

  In this approach the invariant is the spread of the base tranch:

$$\text{Spread}_{Bespoke}(A_{Bespoke}, \rho_{Index}) = \text{Spread}_{Index}(A_{Index}, \rho_{Index}) \tag{12}$$

This approach takes into account the dispersion. A second advantage is simplicity. There are however three immediate disadvantages of this approach. First, it does not take into acount the size of the portfolio neither its duration. Second, one might not find appropriate tranches on the index that would match the spreads in the bespoke. E.g. two examples of are the following: the bespoke portfolio has very low spreads and one needs to price a very senior tranch, in case the bespoke portfolio has too wide tranches and one needs to price a low mezzanine tranche. A third, it does not take into account the size of the tranch.

- **Base Spread Weighted by Tranch Size**

  In this approach the invariant is the base spread received taking into the base tranch size:

$$\text{Spread}_{Bespoke}(A_{Bespoke}, \rho_{Index}) \cdot A_{Bespoke} = \text{Spread}_{Index}(A_{Index}, \rho_{Index}) \cdot A_{Index} \tag{13}$$

  This approach recognises the importance of the tranch size for the mapping technique. It does not however take into account the duration of the contract. The next approach will take into account spread, tranch size and duration.

- **Base Expected Loss at Maturity Matching**

  In this approach one uses the *base expected loss* at maturity (as described in Garcia and Goossens [GG08]) as an invariant between the two portfolios:

$$\frac{\text{BEL}^{Mat}_{Bespoke}(A_{Bespoke}, \rho_{Index})}{\mathbb{E}[L_{Bespoke}]} = \frac{\text{BEL}^{Mat}_{Index}(A_{Index}, \rho_{Index})}{\mathbb{E}[L_{Index}]} \tag{14}$$

  The approach bigest advantage is its intuitiveness as one is dealing directly with the concept of expected loss.

- **Discounted Base Expected Loss Matching**

  This is analogous with the one in (14) but the mapping is done using the discounted expected loss at time zero:

$$\frac{\text{DBEL}^{0}_{Bespoke}(A_{Bespoke}, \rho_{Index})}{\mathbb{E}[L_{Bespoke}]} = \frac{\text{DBEL}^{0}_{Index}(A_{Index}, \rho_{Index})}{\mathbb{E}[L_{Index}]} \tag{15}$$

  The advantages and disadvantages are similar with the ones of the last item.

5 Numerical Results

In this section we show the results of the correlation mapping techniques presented in section 4 in both gaussian copula and Lévy framework.

For illustration and testing purposes we use the CDX.NA.IG as the bespoke portfolio that is mapped into the iTraxx index, both on April 1st, 2008. Table 1 and 2 show detailed results using the base expected loss at maturity and discounted base expected loss (supposed to be more powerfull due to the relation to non-arbitrage conditions) for both the Gaussian copula and Lévy base correlation methodologies. The results are shown in terms of risk duration (RD), base spread, base risk duration (BRD), discounted base expected loss (DBEL), base expected loss at maturity (BELT) and base correlation (BC). The observed market quotes for the CDX index without any mapping are shown as *reference results*.

	Reference Results						
Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
0–3	59.875%	3.417	2252	3.417	2.309	2.544	47.63
3–7	682.5	6.294	792	7.260	4.027	4.584	60.16
7–10	416	7.108	606	8.116	4.914	5.659	65.12
10–15	234	7.757	469	8.284	5.822	6.791	75.34
15–30	101	8.218	284	8.383	7.132	8.479	99.99
	Base Expected Loss Maturity Mapping						
Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
0–3	57.98%	3.525	2145	3.525	2.268	2.503	50.12
3–7	654	6.362	770	7.299	3.934	4.480	62.97
7–10	332	7.381	570	8.197	4.670	5.379	71.02
10–15	185	7.946	432	8.347	5.407	6.311	84.21
15–30	143	8.059	286	8.303	7.132	8.479	99.65
	Discounted BaseExpected Losss Mapping						
Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
0–3	57.68%	3.543	2128	3.543	2.262	2.496	50.53
3–7	640	6.403	761	7.322	3.901	4.444	63.98
7–10	329	7.386	565	8.199	4.631	5.333	71.98
10–15	188	7.934	430	8.343	5.377	6.276	84.83
15–30	145	8.049	286	8.298	7.132	8.479	99.70

Table 1: Gaussian Copula Base Correlation Mapping for CDX.NA.IG on April 1st, 2008

Comparing the spreads after mapping with the refference spreads we see that for the gaussian copula the mapping underprices for all but the [15%–30%] tranche. Additionally the differences can be quite large. For the case of Lévy on the other hand underpricing occurs for two tranches ([3%–7%] and [15%–30%]) and the differences with respect to the referrences are smaller than with the gaussian copula.

Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
	Reference Results						
0–3	59.875%	3.667	2133	3.667	2.346	2.645	73.47
3–7	682.5	6.318	800	7.273	4.071	4.705	70.96
7–10	416	7.098	611	8.112	4.957	5.778	69.35
10–15	234	7.697	473	8.264	5.857	6.880	72.14
15–30	101	8.171	283	8.359	7.095	8.407	97.68
	Base Expected Loss Maturity Mapping						
Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
0–3	58.28%	3.749	2055	3.749	2.311	2.607	74.79
3–7	690	6.290	796	7.258	4.046	4.676	71.51
7–10	337	7.341	585	8.185	4.790	5.579	72.77
10–15	188	7.854	443	8.316	5.530	6.486	79.52
15–30	124	8.075	282	8.311	7.032	8.323	99.37
	Discounted BaseExpected Losss Mapping						
Tr	Spread	RD	Base Spread	BRD	DBEL	BELT %	BC
0–3	58.57%	3.734	2068	3.734	2.317	2.614	74.56
3–7	685	6.303	795	7.265	4.045	4.674	71.53
7–10	333	7.357	584	8.190	4.780	5.567	72.99
10–15	182	7.880	440	8.325	5.498	6.450	80.35
15–30	127	8.062	282	8.305	7.032	8.323	99.37

Table 2: Lévy Base Correlation Mapping for CDX.NA.IG on April 1st, 2008

Fig. 1 and fig. 2 respectively show the Gaussian copula and the Lévy base correlation curves obtained with different mapping techniques. The figures also show the calibrated base correlation curves for iTraxx (the reference index) and CDX (the real curve). Observe that the correlation curves located at the left (right) of the real CDX curve are underpricing (overpricing) equity tranches while overpricing (underpricing) the senior tranches. For the gaussian copula approach the discounted moneyness curve closely follows the real curve. Under the probability approach the generated curve crosses the real curve indicating overvaluation for equity and super senior tranches while potential undervaluation of mezzanine. Observe that all methodologies with the exception of the spread aproach generated convex curves and additionaly there is no smile effect. For the Lévy base correlation approach on the other hand there is a smile. Additionally the crossing of the different curves with the real one happened under all the methodologies.

The expected loss curves and the corresponding base correlation curves for the base expected loss at maturity and the discounted base expected loss are shown in more detail respectivelly in Fig. 3 and Fig. 4. As expected the higher the attachment point the higher the difference between the two approaches. As explained in more detail in Garcia and Goossens [GG08] the interpolation scheme uses cubic splines with the monotonicity filter described in Dougherty et al. [DEH89] applied to it. Observe that for both frameworks the technique preserves the mono-

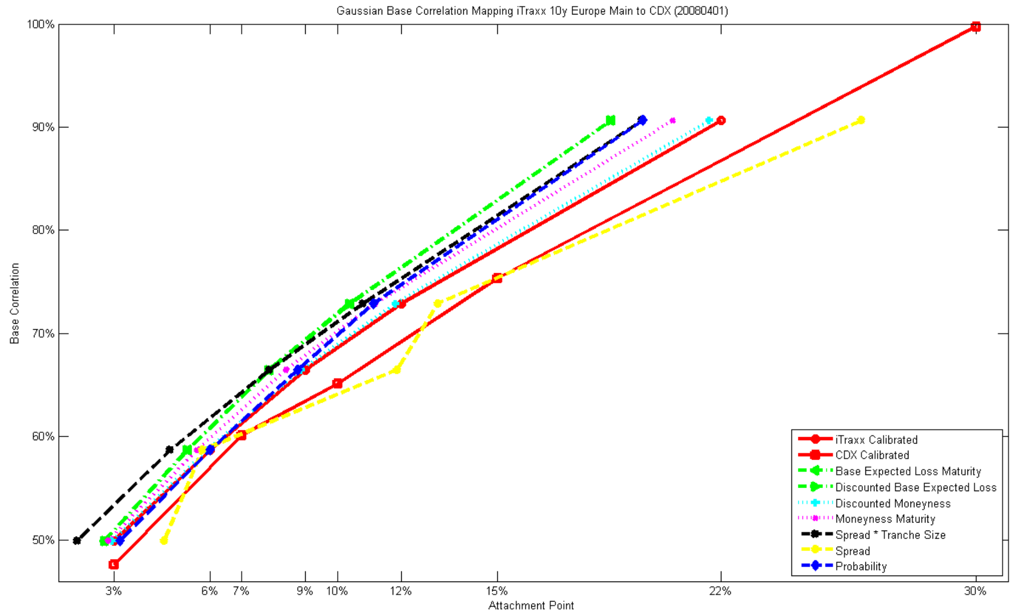

Figure 1: Correlations under the Gaussian Copula Base Correlation Methodology

tonicity and convexity constraints necessary to the absence of arbitrage.

Due to their importance for non-arbitrage conditions Fig. 4 and Fig. 5 depict in more detail the differences of the Gaussian copula and the Lévy base correlation curves obtained with the discounted base expected loss and the base expected loss at maturity.

6 Conclusions

The recent credit crunch that began in Jun 2007 has higlighted the importance of the standardised credit indices for hedging and pricing purposes of portfolios of credit instruments hold by financial institutions. In this paper we have compared the results of several correlation mapping techniques under both gaussian copula and Lévy base correlation frameworks. For the mapping methodologies we have used some traditional techniques such as (discounted) moneyness and probability matching, some of their variances such as moneyness at maturity and approaches based on non-arbitrage opportunities (base expected loss approaches as described in Garcia et al. [GG08]). For academic purposes the CDX index portfolio has been used as a proxy for a bespoke portfolio (whose prices are well known) while the iTraxx portfolio as the index of refference in which the bespoke portfolio is mapped. Results have been shown in terms of mapped correlation curves for the all methodologies. Additionally differences have been highlighted in terms of some useful parameters (for hedging purposes) for the non-arbitrage based approaches. In general we observed quite striking differences between gaussian copula and Lévy based ap-

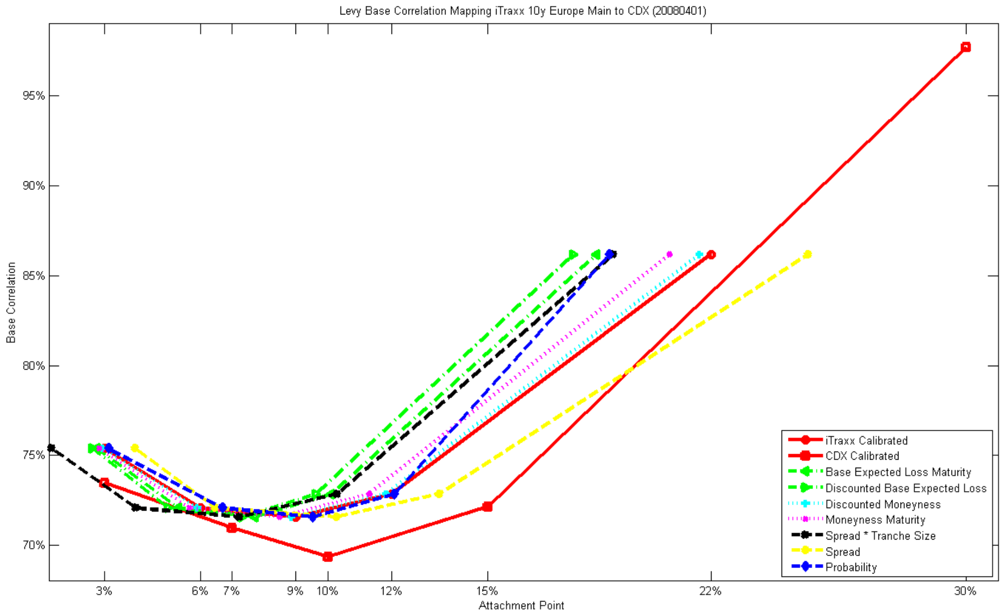

Figure 2: Correlations under the Lévy Base Correlation Methodology

proaches. Additionally prices can be quite sensitive to the correlation mapping technique used. In order to extend the results of this study to the composition of more realistic bespoke portfolios (in here we have used CDX) more extensive studies are necessary and are the current extension of this work.

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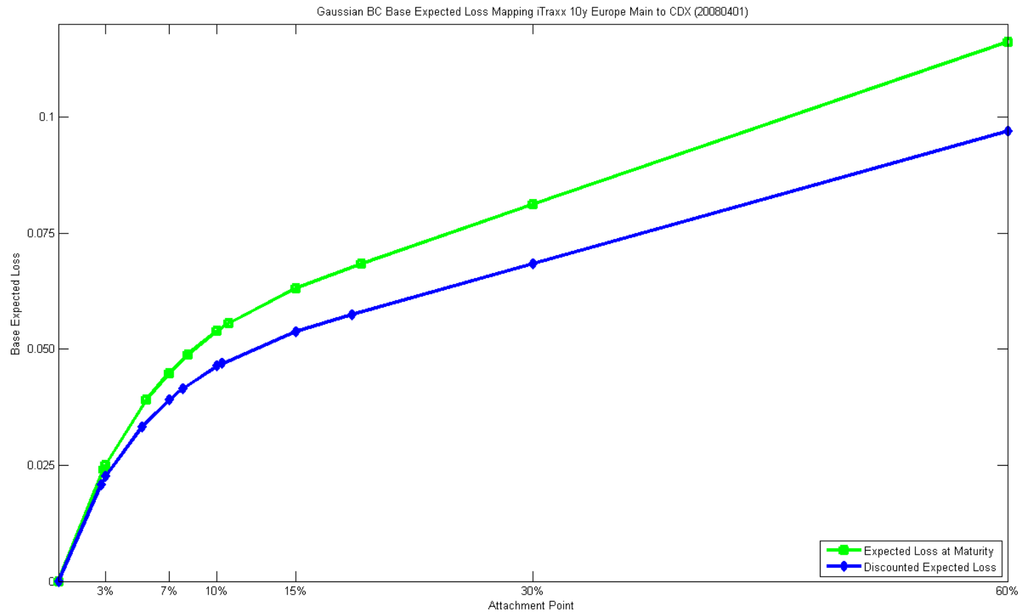

Figure 3: Expected Loss for the BELM and DBELM (for the case of Gaussian Copula)

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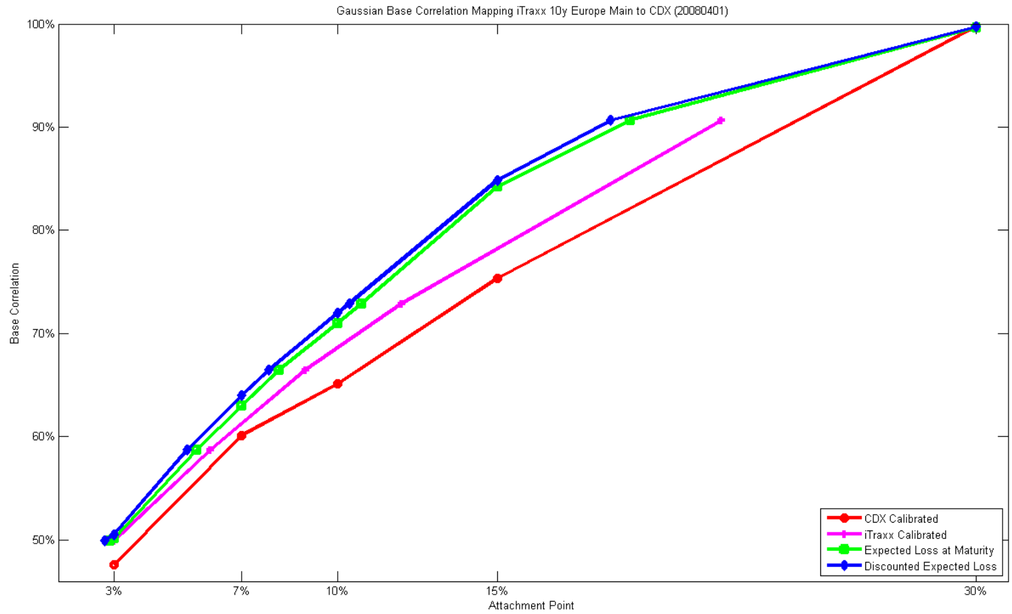

Figure 4: Correlations for the BELM and DBELM (mapping under Gaussian Copula)

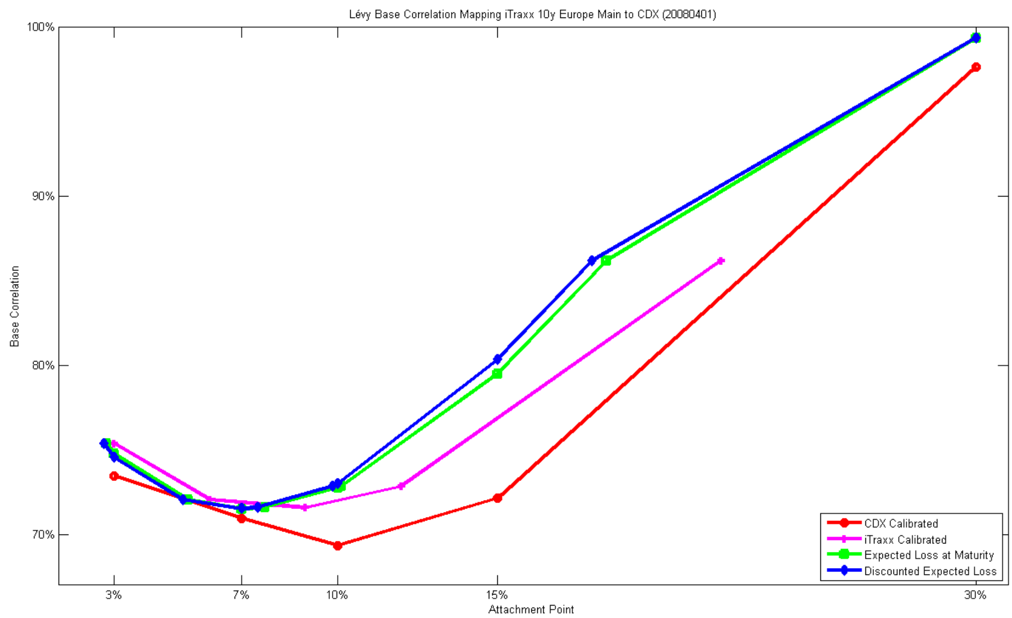

Figure 5: Correlations for the BELM and DBELM (mapping under Gaussian Copula Lévy Base Correlation)