Correlation: from Collateral to ABS Tranches

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Abstract

In this paper we show how variations on the correlation among the underlying collateral of an asset backed security (ABS) impacts the correlation among the tranches in a portfolio of ABS's. Additionally we also show evidence of the dependency on the time frame. This paper should be read in connection with the framework described in Garcia, Goossens and Lamoot [GGL08] as a solution for the securitization business model of financial institution.

1 Introduction

The term *securitization* refers to the pooling and packaging of financial assets in the form of new securities that are sold to investors. Via securitization financial institutions create instruments that can be sold into the market instead of kept on their balance sheets. On the issuer side it improves leverage ratios, the efficient use of capital, and lowers the cost of funding. Additionally it permits the institutions to focus on the business side instead of managing the assets in the balance sheet.

On the investor side securitization permits an investor to have exposure to asset classes that would be impossible otherwise. As an example a pension fund may have access to emerging market (say Latin America bonds) by investing on a senior tranche (AAA rated) of a securitization note. Additionally instead of being exposed to a single name (an idiosyncratic exposure) the investor invests on a diversified portfolio of the asset class.

This has been the securitization business model until June 2007 when the credit crunch brought up additional awareness of the consequences underlying this business model. On one hand the low costs associated with the activity has brought a tremendous economic growth, thus lowering the costs of goods and services while making them more affordable to a larger portion of the population. On the other hand, by substituting single name risk for risk on portfolios, the investor increases the level of systematic risk in its portfolio.

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As has been reported already (see slides 14, 21 and 22 of Garcia [Gar06] for a presentation given *before* the credit turmoil) some market practitioners were well aware of the importance of monitoring the dynamics of correlation of a portfolio of securitization instruments. As described in much more detail in Garcia, Goossens and Lamoot [GGL08] one needs a Dynamic Credit Portfolio Management (DCPM) framework in order to be able to deal with a securitization portfolio. There are two main points underlying the mentioned framework. First, the correlation underlying the ABS notes depends significantly on the holding period. Second, as already observed during the credit turmoil, correlation may change in time. The framework has been designed to serve as a way to monitor the change in correlation. DCPM is a strategic framework to manage the credit *economic capital* of the whole financial institution. Additionally it presents a solution to the problems of variation in correlation of securitization instruments.

More specifically there is an important consequence of the dynamics of correlation underlying ABS's: regulators will certainly increase the cost of capital for those instruments. The longer the holding period, the higher the cost of capital, the larger the impact on the securitization activity. The DCPM framework enters as the solution to keep the cost of capital of securitization instruments to a bare minimum. For accounting purposes, DCPM means that ABS's should be put on the trading portfolios and the standardised credit indices are used to mark to market. The standardised credit indices get a central role on the pricing discovery mechanism as they are supposed to bring liquidity to the instruments. For it transparency in the pricing algorithms and their underlying parameters are key to the whole securitization activity. This key point has already been addressed in earlier papers, see Garcia and Goossens [GG07] for the pricing of TABX tranches.

The current paper is complementary to the articles mentioned above in two ways. First, it shows how changes in correlation of the collateral underlying an ABS affects the correlation between ABS tranches. Second, it shows that this correlation changes with the investment horizon. The paper brings support to the idea that the lowest possible cost of capital is associated with a short term holding period, and as such the securities should be put on trading books and have their values monitored via the standardised credit indices. In the absence of time series of expected loss of ABS tranches we use a Monte Carlo simulation based on the widespread copula algorithm to generate data for the collateral and consequently the prices of the tranches.

The remainder of the paper is organised as follows. In Section 2 we review the generic one factor model used for valuation of CDO tranches and show how to adapt this algorithm for the generation of time series for the current purposes. Numerical results are shown in Section 3. In Section 4 we present the conclusions of the study and wrap it together with the consequences for the current Basel framework and for the securitization business model.

2 Generic One Factor Model

In *latent variable* models default occurs when a certain variable Y_i falls below a threshold K_i which is typically implied from CDS prices. The so called *market* or *systemic* factor X and the *idiosyncratic* factor $X^{(i)}$ are random variables whose functional form depends on model assumptions.

In the generic one factor Gaussian model the latent variable is represented as

$$Y_i = \rho X_m + \sqrt{1-\rho^2}\xi_i \quad (1)$$

where X_m and ξ_i are independent and identically distributed variates, respectively the market factor and the idiosyncratic factor. In the classical Gaussian Copula model, both are standard normal $N(0,1)$ distributed variates. In the generic one factor Lévy model other distributions can be used. Several authors have described one factor models using distributions other than the standard normal distribution. For more details we refer to our earlier paper [GG08a] and the references therein. The approach can easily be extended to a multi factor model, see Garcia and Goossens [GG08b] for the pricing of a CDO of ABS's. Without loss of generality we use a one factor Gaussian copula model in the remainder of this paper.

Assume a portfolio of N firms is considered. We fix a time horizon T. For any $0 \leq t \leq T$ the default times τ_i and default intensities $\lambda_i(t)$, $i = 1, \ldots, N$, satisfy

$$\mathbb{P}(\tau_i > t) = \exp\left(-\int_0^t \lambda_i(u)\mathrm{d}u\right) \quad (2)$$

where $\mathbb{P}$ is the risk-neutral probability measure. The single name survival probabilities $\mathbb{P}(\tau_i > t)$ are typically implied from the credit default swap (CDS) market as follows.

The fair spread of a CDS balances the present value of the contingent leg C, given by

$$C = (1-R)\sum_{i=1}^{n} d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right) \quad (3)$$

and the present values of the fee F leg, given by

$$F = S\left(\sum_{i=1}^{n} P_S(t_i)d(t_i)\Delta t_i + A_D\right), \quad (4)$$

where A_D is the accrual on default

$$A_D = \frac{1}{2}\sum_{i=1}^{n} d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right)\Delta t_i. \quad (5)$$

In these equations the summations run over the payment dates, S is spread premium on a yearly basis, $P_S(t_i)$ is survival probability at time t_i, R is the recovery rate, $d(t)$ is the risk-free discount factor and $\Delta t_i = t_i - t_{i-1}$ is the year fraction.

Typically in a MC simulation one generates random numbers for the market and idiosyncratic factors that are plugged in (1) to generate the latent variable Y_i. Starting from the survival probability distribution implied from CDS quotes, one determines the default time τ_i. The default time τ_i is found by inverting (2) that is by solving for t for a given probability p. In the special case of a constant intensity $\lambda_i(u) = \lambda_i$ we have

$$\tau_i = \frac{-1}{\lambda_i}\log(1 - N(Y_i)), \quad (6)$$

	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
A	41	43	32	20	10	1	59	55	39	26	15	3	75	65	47	35	22	8
B	43	57	50	32	15	1	55	71	62	43	24	4	65	82	72	55	36	12
C	32	50	61	50	27	2	39	62	74	63	38	7	47	72	83	75	51	18
D	20	32	50	57	42	3	26	43	63	72	55	11	35	55	75	83	67	24
E	10	15	27	42	53	11	15	24	38	55	74	30	22	36	51	67	87	48
F	1	1	2	3	11	27	3	4	7	11	30	62	8	12	18	24	48	84

Table 1: Correlation matrices (values in %) for the tranche losses as a function of the asset correlation for 5%, 10% and 20% (from left to right respectivelly).

where $N(x)$ is the standard normal cumulative distribution. If the default time of a certain CDS in the portfolio is lower than its maturity date then the contract is terminated and the outstanding notional is substituted by its recovery value. The contract will go until maturity otherwise.

For each simulation once the losses and their timing are determined for each collateral name one evaluates the losses for each tranche at each point in time and discount it back for the expected loss per simulation. The expected losses of the tranches are given by the average of the expected losses of the simulations made.

3 Numerical Results

In this section we show the simulation results for the correlation between different tranches of a CDO given the asset correlation between the underlying collateral. The collateral return has been generated using the one factor gaussian copula model described in section 2.

In the simulations we were inspired by assumptions used for the corporate standardised credit indices (CDX and iTraxx) both in number of underlying collateral and recovery rates. That is, we considered two baskets of 125 equally weighted names, with a recovery rate of $R = 40\%$.

For simplicity the CDS quotes were distributed as follows, 20% at 50 bp, 20% at 100 bp, 20% at 150 bp, 20% at 200 bp and 20% at 250 bp. Additionally the interest rate curve is assumed to be flat at $r = 4\%$ and the maturity is $T = 5$ year. We considered tranches that match the attachment and detachment points of the iTraxx standardised index: [0 – 3] (A), [3 – 6] (B), [6 – 9] (C), [9 – 12] (D), [12 – 22] (E) and [22 – 100] (F).

Table 1 shows the correlation among the different tranches for a maturity of 5 years the cases of 5%, 10% and 20% asset correlations (from left to right respectively). As expected "similar" tranches (from different deals) are quite correlated, that is the diagonal elements are larger than off-diagonal. The largest moves with respect to a change in asset correlation occur in the case of the super senior tranches.

In Table 2 we show the tranche correlations for the case of 1 year maturity and 5%, 10% and 20% asset correlation. For a given asset correlation comparing the results in Table 1 and Table 2, we see that for all the tranches but the equity all the correlations among the tranches had a significant reduction. Observe that the largest variation occurs for super senior tranche.

	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
A	44	28	7	1	0	0	60	40	16	6	2	0	74	53	29	16	8	1
B	28	37	18	5	1	0	40	58	39	17	6	0	53	75	58	36	18	3
C	7	18	22	12	4	0	16	39	50	35	15	1	29	58	72	59	33	6
D	1	5	12	14	8	0	6	17	35	44	30	2	16	36	59	70	52	10
E	0	1	4	8	10	1	2	6	15	30	44	9	8	18	33	52	74	31
F	0	0	0	0	1	3	0	0	1	2	9	31	1	3	6	10	31	69

Table 2: Correlation matrices (values in %) for the tranche losses as a function of the asset correlation for 5%, 10% and 20% (from left to right) for the 1 year maturity contract.

	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
A	62	52	31	17	8	1	59	59	48	37	25	7	65	66	63	57	46	18
B	52	70	57	35	16	2	59	71	64	49	30	8	66	71	68	58	43	16
C	31	57	69	55	30	3	48	64	75	67	44	11	63	68	75	70	50	18
D	17	35	55	64	47	6	37	49	67	75	60	16	57	58	70	77	64	23
E	8	16	30	47	66	22	25	30	44	60	80	38	46	43	50	64	83	46
F	1	2	3	6	22	52	7	8	11	16	38	69	18	16	18	23	46	76

Table 3: Correlation matrices (values in %) for the tranche losses as a function of the maturity of the contract for 3, 7 and 10 year (from left to right respectively) maturities for an asset correlation of 10%.

The impact on the correlation for the super senior position is more significant when the systemic correlation is lower.

In Table 3 we extend the results of the Table 1 to encompass 3, 7 and 10 years maturity. We see that indeed the longer the maturity the higher the correlation.

Finally Table 4 shows the impact of overlapping the collateral portfolio for 20%, 40% and 60% for a 5 year maturity with 10% asset correlation.

As evidenced during the recent market turmoil the securitization activity is very important for the economic growth of the whole world. Given that correlation determines the cost of capital for a certain instrument, there are very important consequences of the results on the tables shown in this section. First, correlation between tranches of ABS's change depending on asset correlation of the collateral portfolio. Increasing the level of asset correlation increases the level of correlation between tranches. Additionally two ABS's with overlapping collateral will be more correlated than in the absence of overlapping collateral portfolio. Second, *the longer the holding period of an ABS instrument, the larger its cost of capital*. Third, as evidenced by the current credit crunch, the cost of capital of a securitization instrument may dramatically change in time depending on the level of systemic risk. This means that keeping the cost of capital of a securitization instrument fixed (as done in Basel II) may certainly misprice it. Consequently the cost of capital for an instrument should be portfolio dependent!

	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
A	65	58	40	26	15	3	73	61	40	26	15	3	81	64	41	27	15	3
B	58	76	64	44	24	4	61	81	67	44	24	4	64	87	69	45	24	4
C	40	64	78	66	39	7	40	67	82	69	39	7	41	69	87	71	40	7
D	26	44	66	76	58	11	26	44	69	81	59	11	27	45	71	86	61	11
E	15	24	39	58	79	31	15	24	39	59	84	31	15	24	40	61	89	33
F	3	4	7	11	31	68	3	4	7	11	31	73	3	4	7	11	33	81

Table 4: Correlation matrices (values in %) for the tranche losses as a function of the fraction of overlapping underlying assets for an overlap of 20%, 40% and 60% (from left to right respectivelly) for a 5 year contract and an asset correlation of 10%.

In order to attain the lowest cost capital for ABS's the instruments should be put in the trading books, in this case the correlation used in the evaluation of cost of capital would be quite lower in comparison with a long term buy and hold strategy. As those portfolios need to be marked to market another important consequence is that one needs standardised credit indices to be used in the pricing discovery process. As the indices become key instruments for credit portfolio management purposes, the industry needs to have in place transparent (in terms of model and input parameters) and fast pricing algorithms for all the standardised credit indices.

The issue of transparency in the pricing process of standardised credit indices has been treated in much detail in an earlier paper, see Garcia and Goossens [GG07]. Although the pricing methodologies for the corporate credit indices were already in place since long before the credit crunch that began in Jun 2007, the same can not be said for the index (and its tranches) of subprime MBS's (ABX and TABX respectively). In the study mentioned earlier we have adapted the standard one factor gaussian copula model using the standard recursive approach of Andersen, Sidenius and Basu [ASB03] to price tranches of TABX.BBB-. Using prepayment assumptions underlying the cash instruments it is impossible to find any level of correlation that would match observed market prices. The reason for adapting the well known recursive approach for pricing purposes of the TABX indices is to be able to use the same correlation mapping techniques currently in use for the corporate bespoke CDO's to price bespoke tranches of CDO's of ABS's. The proposed approach does not preclude one to use the more traditional MC based approach where the particularities of the each deal are modeled in detail. We point out however the necessity of a *fast* and *transparent* methodology for pricing indices for trading and hedging purposes.

The securitization business model is very important for financial institutions and the economy as a whole. As the tables above prove the correlations on those instruments can vary significantly. If the positions are put on buy and hold on the portfolios of a financial institution, the correlation to be used might be inhibitive for the activity. Putting the positions in trading brings the cost of capital for the activity significantly lower. Additionally it brings transparency to the books of a financial institution. The necessity of having costs of capital to be portfolio dependent brings the necessity of a sofisticated system capable of handling large portfolios. A proposal that puts all the mentioned issues together to the benefit of the securitization business model has been outlined in the context of a Dynamic Credit Portfolio Management framework for the strategic management of a financial institution by Garcia, Goossens and Lamoot [GG08b]. In a forthcoming paper we will show how the results of this paper can be put together with the framework referred for the dynamic management of a credit portfolio.

4 Conclusions

In Garcia, Goossens and Lamoot [GGL08] we have presented a framework to dynamically manage the credit portfolio of a financial institution as a solution to the securitization business model. The reason behind the framework is that correlation between tranches of ABS's changes in time depending on the level of systemic correlation in the underlying portfolio. This study brings the evidence that indeed tranche correlation can change significantly due to changes in the systemic risk of the underlying portfolio. We also show how the tranche correlations change with maturity and asset correlation of underlying collateral. Additionally we show the impact of increases in overlapping on the collateral portfolio. We also call the attention of the reader to the necessity of transparency on the pricing methodologies and input parameters. The dynamic of correlation affects costs of capital for securitization instruments increasing this cost in case the instruments are on hold to maturity, potentially impacting the securitization activity and the whole economy. Lower cost of securitization can come if the positions are put on the trading books and are marked to market using the standardised credit indices. The indices become key components on the pricing discovery process of securitization portfolios and may certainly be used as early warning signals of possible market deterioration. This can be seen for example in the increase on the correlation for super senior tranches. In a forthcoming paper we will put the results presented here together with the mentioned Dynamic Credit Portfolio Management framework for the management of a credit portfolio.

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