Let's Jump Together
Pricing of Credit Derivatives: From Index Swaptions to CPPIs

João Garcia*, Serge Goossens†, and Wim Schoutens‡

May 8, 2007

*Dexia Group, Galileïlaan 5, B-1210 Brussels, Belgium, E-mail: crisj@dexia.com

†Dexia Bank, Pachecolaan 44, B-1000 Brussels, Belgium, E-mail: Serge.Goossens@dexia.be

‡K.U.Leuven, Department of Mathematics, Celestijnenlaan 200 B, B-3001 Leuven, Belgium. E-mail: wim@schoutens.be

Abstract

This paper describes a dynamic multivariate jump driven model in a credit setting. We set up a dynamic Lévy model, more precisely a Multivariate Variance Gamma (VG) model, for a series of correlated spreads. The parameters of the model come from a two step calibration procedure. First, a joint calibration on swaptions on the spreads is performed and second, a correlation matching procedure is applied. For the first calibration step, we make use of equity-like pricing formulas for payer and receiver swaptions, based on the characteristic function and the Fast Fourier Transform (FFT) method. In the second calibration step, we fix the correlation in the model to match the prescribed (in casu historically observed) correlation. This can be done fast since a closed form expression is readily available. The resulting jump driven dynamic model generates correlated spreads very fast. This model can be used to price a whole range of exotic structures. We illustrate this by pricing the currently popular credit Constant Proportion Portfolio Insurance (CPPI) structures. Because of the built in jump dynamics a better assessment of gap risk is possible.

1 Introduction

In recent years we have seen a huge growth of structured credit derivatives. One such innovation is a credit Constant Proportion Portfolio Insurance (CPPI). A credit CPPI is capital guaranteed (principal protected) investment strategy and in its simplest format works just like a capital protected credit linked note (CLN). The invested capital is put in a risk free bond and a position is taken on credit derivatives. Usually protection is sold on a basket or on standardized corporate Collateralized Debt Obligation (CDO) indices such as iTraxx and CDX. Intrinsic to the price of a CPPI is the risk of spread jumps of the underlying credit indices on which the positions are taken, this is the so called gap risk.

A second recent innovation is the possibility to trade options on indices. The importance of a liquid option market cannot be underestimated. Suppose a bank enters into a loan and into a Credit Default Swap (CDS) on the loan. In case the loan is prepaid the bank will want to be able to cancel the CDS (a cancelable CDS). Another example of the necessity for options is is the hedge of a credit CPPI. Ideally the gap risk would be hedged by buying an out of the money call on an index CDO. Additionally the calibration of any dynamic spread model involves pricing European options on CDSs.

In this paper we build Lévy based dynamic spread models to price index swaptions and CPPI structures. We set up a dynamic Lévy model, more precisely a Multivariate Variance Gamma (VG) model, for a series of correlated spreads. The parameters of the model come from a two step calibration procedure. First, a joint calibration on swaptions on the spreads is performed and second, a correlation matching procedure is applied. For the first calibration step, we make use of equity-like pricing formulas for payer and receiver swaptions, based on the characteristic function and the Fast Fourier Transform (FFT) method. In the second calibration step, we fix the correlation in the model to match the prescribed (in casu historically observed) correlation. This can be done fast since a closed form expression is readily available. The resulting jump driven dynamic model generates correlated spreads very fast. This model can be used to price a whole range of exotic structures. We illustrate this by pricing the currently popular credit Constant Proportion Portfolio Insurance (CPPI) structures. Because of the built in jump dynamics a better assessment of gap risk is possible.

2 Multivariate Variance Gamma Modeling

2.1 Multivariate Variance Gamma Processes

Let us start with the univariate Variance Gamma $VG(\sigma, \nu, \theta)$ distribution, which is defined by its characteristic function

$$\phi(u) = \left(1 - iu\theta\nu + \frac{1}{2}\sigma^2\nu u^2\right)^{-1/\nu}.$$

This distribution dates back to the early work of Madan and Seneta ([MS87] and [MS90]). The Variance Gamma (VG) distribution is known to be infinitely divisible and by standard Lévy theory one thus can build out of such a distribution a process, $X = \{X_t, t \geq 0\}$ with parameters $\sigma, \nu > 0$ and θ, which has stationary and independent VG increments (for more details see [Sch03]). Theoretical background on Lévy process can be found in [Ber96] and [Sat00]. For some historical background on the VG model see [Sen07].

Another way of constructing a VG process is by the technique of time changing. We use this technique later on to introduce the multivariate VG process. We start with defining the Gamma process. Recall that the density function of a Gamma(a, b) distribution is given by

$$f_{\text{Gamma}}(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-xb), \quad x > 0,$$

where $\Gamma(\cdot)$ is the Gamma function. The Gamma distribution is also known to be infinitely divisible and again by standard Lévy theory, one can build out of such a distribution a process, with stationary and independent gamma increments. So, consider a Gamma process $G = \{G_t, t \geq 0\}$ with parameters $a = b = 1/\nu$. This actually means that G_t follows a Gamma(at, b) distribution and $E[G_t] = t$. A VG process can be constructed by time-changing a Brownian Motion with drift. More precisely, one can show (for more details see e.g. [Sch03]) that the process

$$X_t = \theta G_t + \sigma W_{G_t}, t \geq 0,$$

where $W = \{W_t, t \geq 0\}$ is a standard Brownian motion independent from the Gamma process, is indeed a VG process with parameters (σ, ν, θ). The construction can be interpreted as looking at a Black-Scholes world but now measured according to a new business clock (Gamma time). It has proven to be very successful in the univariate setting, as the underlying VG distribution can take into account, in contrast to the Normal distribution, skewness and excess-kurtosis. In for example [CW04], a VG model was used to accurately fit CDS curves.

Additionally, the paths are now completely jump driven. A summary of some characteristics of the distribution can be found in Table 1.

	VG(σ, ν, θ)	**VG**$(\sigma, \nu, 0)$
mean	θ	0
variance	$\sigma^2 + \nu\theta^2$	σ^2
skewness	$\theta\nu(3\sigma^2 + 2\nu\theta^2)/(\sigma^2 + \nu\theta^2)^{3/2}$	0
kurtosis	$3(1 + 2\nu - \nu\sigma^4(\sigma^2 + \nu\theta^2)^{-2})$	$3(1 + \nu)$

Table 1: Characteristics of the Variance Gamma distribution.

.

We work with multivariate extensions of the model along the technique described in [LS06]. Additionally, we want to note that in [MS90], a symmetric version of a multivariate VG is initiated.

To build the multivariate Variance Gamma model (MVG), we need several ingredients. We need once more a Gamma process $G = \{G_t, t \geq 0\}$ with parameters $a = b = 1/\nu$. Now take a N-dimensional Brownian motion that we denote as

$\vec{W} = \{\vec{W}_t = (W_t^{(1)}, \dots, W_t^{(N)}), t \geq 0\}$. We assume that $\vec{W}$ is a process that is independent of the Gamma process $G = \{G_t, t \geq 0\}$ and that the Brownian motions have a correlation matrix given by $\rho^W = (\rho_{ij}^W, i, j = 1 \dots N)$:

$$\rho_{ij}^W = E[W_1^{(i)} W_1^{(j)}].$$

A multivariate VG process $\vec{X} = \{\vec{X}_t = (X_t^{(1)}, \dots, X_t^{(N)}), t \geq 0\}$ is defined as:

$$X_t^{(i)} = \theta_i G_t + \sigma_i W_{G_t}^{(i)}.$$

Note that there is dependence between the $X_t^{(i)}$'s due to two causes: they are all constructed by a time-change with a common Gamma time. This will mean that the processes will all jump together, but jumps' sizes can be different. Moreover there is dependency also built in via the Brownian motions. A straightforward calculation shows that the correlation between two components is given by:

$$\begin{aligned} \rho_{ij} &= \frac{E[X_1^{(i)} X_1^{(j)}] - E[X_1^{(i)}]E[X_1^{(j)}]}{\sqrt{\mathrm{Var}[X_1^{(i)}]}\sqrt{\mathrm{Var}[X_1^{(j)}]}} \\ &= \frac{\theta_i \theta_j \nu + \sigma_i \sigma_j \rho_{ij}^W}{\sqrt{\sigma_i^2 + \theta_i^2 \nu}\sqrt{\sigma_j^2 + \theta_j^2 \nu}}. \end{aligned} \tag{1}$$

This clearly shows that, even if we assume the Brownian components to be independent of each other, one still obtains a correlation between the different components because of the common time-clock. In fact, the parameter ν introduces a common time-change for all the assets. It can be seen as a background parameter that lets time evolve faster or slower, depending on the market conditions; all the assets experience the same stochastic clock.

2.2 Multivariate Variance Gamma Spread Dynamics

Suppose, we need to model, as later on will be the case, the evolution of N correlated spreads. Take for example the evolution of the iTraxx Europe main index and its overseas relative the Dow Jones CDX.NA.IG main index and the spreads of their respective HiVol subsets.

We assume the following correlated dynamics for the evolution of N dependent spreads:

$$S_t^{(i)} = S_0^{(i)} \exp(\omega_i t + \theta_i G_t + \sigma_i W_{G_t}^{(i)}) = S_0^{(i)} \exp(\omega_i t + X_t^{(i)}), \quad i = 1, \dots, N,$$

where

$$\omega_i = \nu^{-1} \log\left(1 - \frac{1}{2}\sigma_i^2 \nu - \theta_i \nu\right).$$

These mean correction terms, ω_i are in place because one can then easily show that the spread processes are mean reverting in the sense that we have for every $t \geq 0$

$$E[S_t^{(i)}] = S_0^{(i)}.$$

3 Swaptions on Credit Indices

Two types of index swaptions are currently traded: payers and receivers. All index swaptions are European style. A payer/receiver option holder has on expiry the right but not the obligation to buy/sell protection on the underlying index at the strike level. If a default happens among the index constituents prior to option expiration, both the buyer of a payer or the seller of a receiver option can trigger on expiry a credit event (so called non-knockout feature).

The payoff of an index swaption at expiry has two components: payoff due to difference between expiry spread level and the strike and payoff due to any default losses. For short-maturity options the later is very unlikely to happen and is often ignored.

Let us introduce some notation. Denote by T the payer or receiver swaption maturity, typically 3, 6, or 9 months, and by T^* the index maturity, typically 5, 7 or 10 years. Let us denote with A_t the risky annuity for maturity t, that is the present value of 1 basis point (bp) of the fee leg. The forward annuity is denoted by $A(T, T^*)$ and is the forward annuity from swaption maturity to index maturity as of the trade day ($t = 0$). We have of course that $A_t = A(0, t)$ and $A(T, T^*) = A_{T^*} - A_T$.

The forward spread as of the trade day, that is at time $t = 0$, $F_0 = F_0(T, T^*)$, is the forward spread from swaption maturity T to index maturity T^* and is given by

$$F_0(s, t) = \frac{S_t A_t - S_s A_s}{A_t - A_s}.$$

3.1 Black's Model

The market standard for modeling credit spread options is a modification of Black's formula for interest rate swaptions (see e.g. Pederson [Ped04]). It models spread dynamics as

$$S_t = S_0 \exp(-\sigma^2 t/2 + \sigma W_t), S_0 > 0,$$

where $\sigma > 0$ is the volatility parameter.

Black's formula is given by

$$\begin{aligned} \text{Payer}(T, K) &= A(T, T^*)(F_0 \text{N}(d_1) - KN(d_2)) \\ \text{Receiver}(T, K) &= A(T, T^*)(KN(-d_2) - F_0 \text{N}(-d_1)), \end{aligned}$$

where

$$d_1 = \frac{\log(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

If the payer swaption is non-knockout, as is typically the case for the index swaptions we are dealing with, we adjust the forward spread to account for the non-knockout feature of index options. We account for this additional protection by increasing the forward spread by the cost of this protection. More precisely, the adjusted forward spread is given by

$$F_0^{(adj)} = F_0^{(adj)}(T, T^*) = F_0(T, T^*) + \frac{S_{T^*} A_T}{A_{T^*} - A_T},$$

and the price of non-knockout payers and receivers is resp. given by

$$\begin{aligned}\text{Payer}(T,K) &= A(T,T^*)(F_0^{(adj)}\text{N}(d_1) - KN(d_2)) \\ \text{Receiver}(T,K) &= A(T,T^*)(KN(-d_2) - F_0^{(adj)}\text{N}(-d_1)),\end{aligned}$$

where

$$d_1 = \frac{\log(F_0^{(adj)}/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

Note the striking connection with vanilla option prices in equity. Basically spread option pricing comes down to pricing under a Black-Scholes regime with no interest rates and no dividends. However the model has all the deficiencies of the Black-Scholes framework: no-jumps, light-tails, symmetric underlying distribution, and so on.

3.2 Variance Gamma Model

Our approach is completely similar to the equity setting. We replace the Black-Scholes dynamics with the better performing jump dynamics of VG. We now model the spread dynamics as

$$S_t = S_0 \exp(\omega t + \theta G_t + \sigma W_t) = S_0 \exp(\omega t + X_t).$$

Pricing of vanilla options has already been worked out in full detail in equity settings [CM98] and by a slight adaption to the credit setting we can very fast calculate payer and receiver swaptions using the Carr-Madan formula in combination with FFT methods. More precisely,

$$\text{Payer}(T,K) = A(T,T^*)\frac{\exp(-\alpha\log(K))}{\pi}\times$$
$$\int_0^{+\infty} \exp(-\mathrm{i}v\log(K))\frac{\phi(v-(\alpha+1)\mathrm{i};T)}{\alpha^2+\alpha-v^2+\mathrm{i}(2\alpha+1)v}\mathrm{d}v,$$

where the characteristic function of the logarithm of the adjusted forward spread process at maturity T is given by

$$\phi(u;T) = E\left[\exp\left(\mathrm{i}u\left(\log F_0^{(\text{adj})} + \omega T + X_T\right)\right)\right],$$

which is known analytically in many Lévy settings, including Variance Gamma.

4 Correlated Variance Gamma Spread Processes

We now have a multivariate spread model available and fast pricers for the standard payer and receiver swaptions. Using these fast pricers on index swaption, we calibrate our model in a two step procedure. First, we make sure that our model reproduces the swaption market data by doing a joint calibration using our fast FFT pricer. Second, we put in place the exact correlation structure we want, by calculating using a closed-form formula the correlation matrix of the underlying driving standard Brownian motions. We illustrate the use of the model by a working out an example, where we price a CPPI structure.

4.1 Constant Proportion Portfolio Insurance Pricing

The Constant Proportion Portfolio Insurance (CPPI) was first introduced by Fisher Black and Robert Jones in [BJ87]. Recently, credit-linked CPPIs have become popular to create capital protected credit-linked notes. CPPI products are leveraged investments whose returns depend on the performance of an underlying trading strategy. Quite often positions are taken on the available credit indices (iTraxx and CDX). Credit CPPIs combine dynamic leverage with principal protection. Leverage is increased when the strategy performs well and is reduced when it performs poorly. We work out the details of such a product as an example, where positions will be taken in 4 highly correlated indices and a predefined trading strategy is in place.

In our example, we take positions in the following index products:

- iTraxx Europe Main On the run (5Yr) Unfunded
- iTraxx Europe HiVol On the run (5Yr) Unfunded
- DJ CDX.NA.IG Main On the run (5Yr) Unfunded
- DJ CDX.NA.IG High Volatility On the run (5Yr) Unfunded

The swap rates for the above products are highly correlated as can be seen from Figure 1.

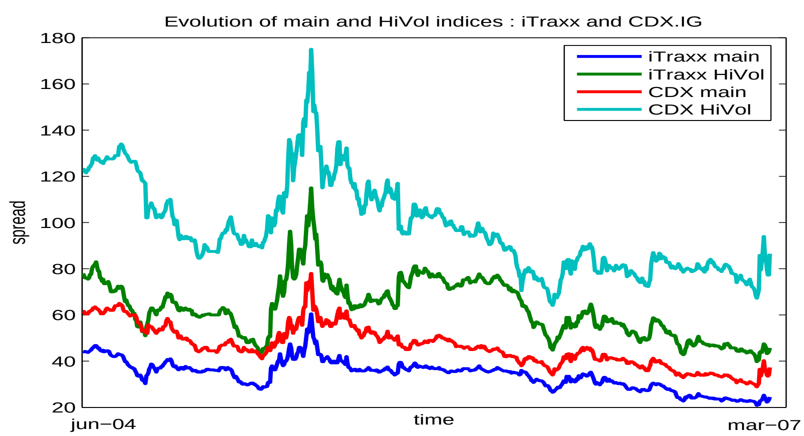

Figure 1: Evolution of Main and HiVol indices: iTraxx and CDX.NA.IG.

The corresponding correlation matrix of spreads itself and the correlation matrix of the corresponding daily log returns based on observations from the 21st of June 2004 until the 13th of March 2007 are given in Tables 2 and 3, respectively.

We start with a portfolio of say 100M EUR and an investment horizon of 6 years. We want to have the principal of the initial investment protected. Therefore we calculate the bond floor as the value of a risk-free bond that matures at the end of the

correlation	iTtraxx Main	iTraxx HiVol	CDX Main	CDX HiVol
iTtraxx Main	1.0000	0.9162	0.9194	0.8469
iTraxx HiVol	0.9162	1.0000	0.7687	0.7580
CDX Main	0.9194	0.7687	1.0000	0.9446
CDX HiVol	0.8469	0.7580	0.9446	1.0000

Table 2: Correlation of spreads: iTraxx (main and HiVol) and CDX (main and HiVol).

correlation	iTtraxx Main	iTraxx HiVol	CDX Main	CDX HiVol
iTtraxx Main	1.0000	0.9258	0.4719	0.3339
iTraxx HiVol	0.9258	1.0000	0.4398	0.3281
CDX Main	0.4719	0.4398	1.0000	0.8580
CDX HiVol	0.3339	0.3281	0.8580	1.0000

Table 3: Correlation of log-returns: iTraxx (main and HiVol) and CDX (main and HiVol).

investment horizon. Suppose we take $r = 0.04$ and use compound interest rates, then the bond-floor is initially at 78.6628M EUR. Suppose we set the leverage at $m = 30$. The cushion is defined as the difference between the portfolio value and the bond-floor. Initially, the cushion is thus at 21.3372M EUR. Multiply the cushion with the constant leverage factor of 30, gives the risky exposure that we are going to take, namely 640.12M EUR. We are taking the following positions:

- sell protection on iTraxx Europe Main On the run (5Y) for half of the risky exposure,
- buy protection on iTraxx Europe HiVol On the run (5Y) for $\frac{1}{10}\frac{1}{2}\frac{30}{125}$ the risky exposure,
- sell protection on DJ CDX.NA.IG Main On the run (5Yr) for half of the risky exposure, and
- buy protection on DJ CDX.NA.IG HiVol On the run (5Yr) for $\frac{1}{10}\frac{1}{2}\frac{30}{125}$ of the risky exposure.

The initial 100M EUR is put on a risk-free bank account at a compound rate of 4 percent. The initial quotes for the four components of our portfolio are given in Table 4.

We rebalance regularly, e.g. daily. We continue doing this until maturity or until we have at a rebalancing date a negative cushion. In that case all positions are closed. We can however not pay back the principal amount between since the portfolio value is below the bond-floor. This is called gap risk (see Figure 2). One of the aims of the model is to calculate the gap risk.

	$t = 0$
iTtraxx Main	24.625
iTraxx HiVol	48.75
CDX Main	37.5
CDX HiVol	88.5

Table 4: Initial quotes for the four components of our portfolio in bp.

We assume the following correlated VG dynamics for the spreads:

$$\begin{aligned}
S_t^{(1)} &= S_0^{(iTraxxMain)} \exp(\omega_1 t + \theta_1 G_t + \sigma_1 W_{G_t}^{(1)}) \\
S_t^{(2)} &= S_0^{(iTraxxHiVol)} \exp(\omega_2 t + \theta_2 G_t + \sigma_2 W_{G_t}^{(2)}) \\
S_t^{(3)} &= S_0^{(CDXMain)} \exp(\omega_3 t + \theta_3 G_t + \sigma_3 W_{G_t}^{(3)}) \\
S_t^{(4)} &= S_0^{(CDXHiVol)} \exp(\omega_4 t + \theta_4 G_t + \sigma_4 W_{G_t}^{(4)}),
\end{aligned}$$

where G_t is a common Gamma Process, such that $G_t \sim \text{Gamma}(t/\nu, 1/\nu)$, and $W_t^{(i)}$ are correlated standard Brownian motions with a given correlation matrix $\rho^W = (\rho_{ij}^W)$.

Under the model log-returns have the following correlation as in Equation (1):

$$\begin{aligned}
\rho_{ij} &= \frac{E[\log S_1^{(i)} \log S_1^{(j)}] - E[\log S_1^{(i)}]E[\log S_1^{(j)}]}{\sqrt{\text{Var}[\log S_1^{(i)}]}\sqrt{\text{Var}[\log S_1^{(j)}]}} \\
&= \frac{\theta_i \theta_j \nu + \sigma_i \sigma_j \rho_{ij}^W}{\sqrt{\sigma_i^2 + \theta_i^2 \nu}\sqrt{\sigma_j^2 + \theta_j^2 \nu}}.
\end{aligned}$$

We will first perform a joint calibration on swaptions of the individual indices. This determines the parameters ν and θ_i and σ_i, $i = 1, \ldots, 4$. Next, we match the historical correlations with ρ_{ij} by setting

$$\rho_{ij}^W = \frac{\rho_{ij}\sqrt{\sigma_i^2 + \theta_i^2 \nu}\sqrt{\sigma_j^2 + \theta_j^2 \nu} - \theta_i \theta_j \nu}{\sigma_i \sigma_j}.$$

Hence we are able to match quite accurately all the individual spread dynamics by correlated jump processes and moreover are able of imposing a correlation structure completely matching observed historical correlation. Indeed for our working example, the result of the calibration can be found in Figure 3. In order to match the required correlation, which we have taken from the log-return historical correlation from 21st June 2004 until 13th March 2007 as shown in Table 3), we need to set the Brownian correlation matrix equal to:

$$\rho^W = \begin{bmatrix} 1.0000 & 0.9265 & 0.4935 & 0.3352 \\ 0.9265 & 1.0000 & 0.4470 & 0.3247 \\ 0.4935 & 0.4470 & 1.0000 & 0.8688 \\ 0.3352 & 0.3247 & 0.8688 & 1.0000 \end{bmatrix}.$$

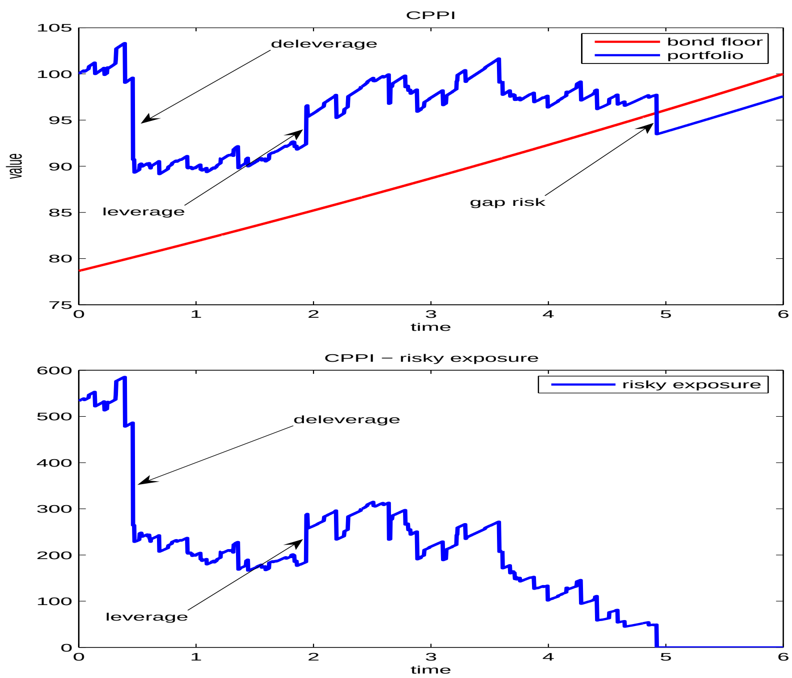

Figure 2: CPPI performance, leveraging, deleveraging and gap risk in a multivariate Variance Gamma driven model.

In Figure 4 one sees a typical picture of the correlated moves under the multivariate VG jump dynamics.

4.2 Gap Risk

The gap risk under Black's model with continuous rebalancing is zero. Due the continuous paths of the Brownian Motion, the bond floor is never crossed but always hit as in Figure 5. One could artificially rebalance only periodically, say quarterly, in order to generate some gap risk. Much more natural and conforming to reality, is to rebalance continuously (or daily), and to assume jump dynamics in the model. Indeed, if jumps are present in the spread dynamics, then the portfolio value also jumps. Its value can suddenly jump below the bond-floor. Hence the price of this gap risk is not zero anymore. The price to cover against the gap risk, estimated based on 100000 Monte Carlo simulations, for the multivariate VG model in our example for instance is around 5 bp per year.

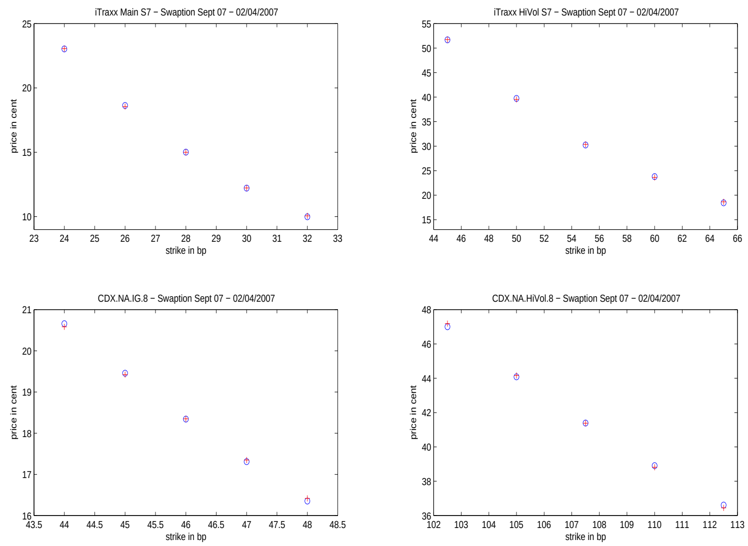

Figure 3: Calibration on swaptions.

5 Conclusion

We have introduced a multivariate jump model to model correlated credit spread indices. The model is calibrated on swaptions on the individual indices. The model correlation can be matched with a given correlation. The pricing of payer and receiver swaptions is done by a modification of the Carr-Madan formula, which was originally introduced to price calls and puts in an equity setting. Monte Carlo simulation of paths under the model is straightforward. This allows us to price and investigate multivariate products or portfolio strategies under a calibrated model. This is illustrated by pricing the gap risk of a credit CPPI product.

Note that the model is not restricted to a pure credit setting, but can equally well be applied in a hybrid setting.

One can set up multivariate VG dynamics where for example equity indices and stock dynamics are combined with credit dynamics. This is possible in case fast option pricers are available under a univariate VG model, like is the case for equity vanilla options and credit swaptions. Hybrid CPPIs or other portfolio products, like Constant Proportion Debt Obligations (CPDO), taking positions in equity and credit can be defined and priced. Typically negative correlation will be in place here.

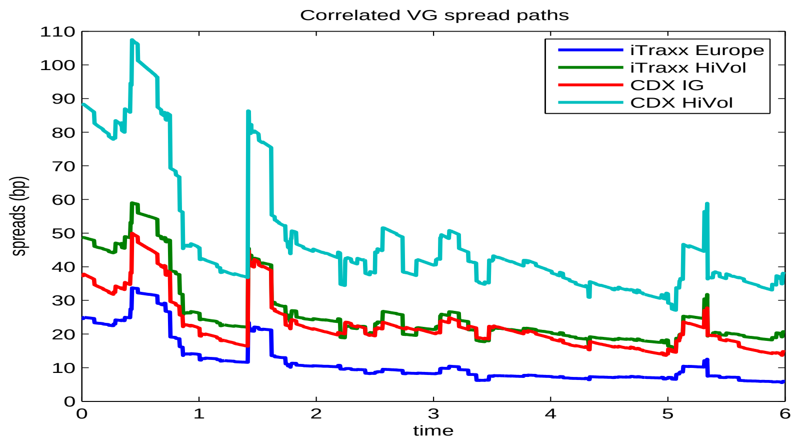

Figure 4: Correlated Variance Gamma spread paths.

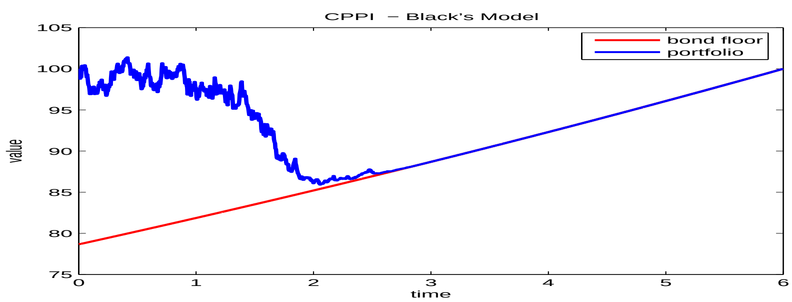

Figure 5: The gap risk is zero under Black's model with continuous rebalancing, due the continuity of Brownian Motion.

References

[Ber96] J. Bertoin. *Lévy Processes*, volume 121 of *Cambridge Tracts in Mathematics*. Cambridge University Press, 1996.

[BJ87] F. Black and R. Jones. Simplifying portfolio insurance. *Journal of Portfolio Management*, 1987.

[CM98] P. Carr and D. Madan. Option valuation using the fast fourier transform. *Journal of Computational Finance*, 2:61–73, 1998.

[CW04] J. Cariboni and Schoutens W. Pricing credit default swaps under lévy models. Technical report, K.U.Leuven, 2004. UCS Report 2005-02.

[LS06] E. Luciano and W. Schoutens. A multivariate jump-driven financial asset model. *Quantitative Finance*, 6(5):385–402, 2006.

[MS87] D.B. Madan and E. Seneta. Chebyshev polynomial approximations and characteristic function estimation. *Journal of the Royal Statistical Society Series B*, 49(2):163–169, 1987.

[MS90] D. Madan and E. Seneta. The v.g. model for share market returns. *Journal of Business*, 63:511–524, 1990.

[Ped04] C.M. Pederson. Introduction to credit default swaptions. Technical report, Lehman Brothers, 2004. Quantative Credit Research 2004, Q3/4.

[Sat00] K. Sato. *Lévy Processes and Infinitely Divisible Distributions*, volume 68 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 2000.

[Sch03] W. Schoutens. *Lévy Processes in Finance: Pricing Financial Derivatives*. John Wiley & Sons, 2003.

[Sen07] E. Seneta. *The Early Years of the Variance-Gamma Process*. Birkhaeuser, 2007. to appear.