# Base Expected Loss explains Lévy Base Correlation Smile

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**Abstract**

In an earlier paper we introduced Lévy base correlation. In this paper we look at base expected loss at maturity both in the Gaussian copula and Lévy based models and link it to base correlation in these frameworks. We report on the existence of smile in both base correlation curves and discuss different interpolation methodologies in view of absence of arbitrage. Finally we discuss the properties of these curves for tranchlet pricing purposes.

## 1 Introduction

There has been an enormous technological innovation behind the incredible growth of the credit derivatives market. The success of the credit derivatives instruments can be seen by the growth in the outstanding notional amount in the market from about 3,000 billion USD back in 2003 to 20,000 billion USD in 2006 (see e.g. [Ass06]).

Since the introduction of the 1-factor Gaussian copula model for pricing synthetic Collateralized Debt Obligation (CDO) tranches by Andersen et al. [ASB03] correlation is seen as an exogenous parameter used to match observed market quotes. First the market adopted the concept of implied compound correlation. In tandem with the concept of volatility in the Black Scholes option pricing framework compound correlation was the parameter to be put in the model to match observed market prices of tranches.

One of the problems of this approach resides in its unsuitability for interpolation. Given the implied compound correlations for the [3%–6%] and [6%–9%] tranches of a liquid index such as iTraxx Europe Main[1] it is not clear which value to use for a nonstandard tranche such as e.g.

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[1]The standard attachment points are 3%, 6%, 9%, 12% and 22% for iTraxx Europe Main and 3%, 7%, 10%, 15% and 30% for CDX.NA.IG.

[5%–8%]. Besides the problems of interpolation we can also mention that during some market events correlation may receive non-meaningful values (e.g. during the auto crisis of May 2005).

The current widespread market approach is to use the concept of base correlation (BC) introduced by McGinty et al. [MBAW04]. In the base correlation methodology only equity or base tranches[2] are defined. The price of a tranche [A - D] is calculated using the two equity tranches with A and D as detachment points. Using BC it is quite straightforward to bootstrap between standard attachment points. Additionally the BC concept is quite adapted to interpolation for nonstandard tranches. Hence the [5%–8%] tranche would be priced by interpolating the BC curve for values at 5% and 8% respectivelly.

The methodology however has some weaknesses. First of all it is very sensitive to the interpolation technique used. Even worse, the methodology may not be arbitrage-free. Finally the methodology does not provide any guidance on how to extrapolate the curve, especially below the 3% attachment point.

Another characteristic of the standard market approach is its reliance on the Gaussian copula. In an earlier paper [GGMS07] we introduced the concept of Lévy base correlation. In this paper we introduce the concept of base expected loss and link it to the Lévy BC concept. The base expected loss concept is inspired by the work of Parcell and Wood [PW07]. Our approach differs from theirs in two important ways. First we use it in the Lévy framework, and second we look at the expected loss at a certain point in time (in this case at maturity).

The remainder of this paper is organised as follows. In section 2 we review the generic 1-factor model for valuation of CDO tranches. In section 3 we discuss interpolation in the base correlation framework. Our Lévy base correlation methodology is detailed in section 4. In section 5 we define base expected loss, describe its properties and outline the upper and lower bounds it must satisfy. In section 6 we discuss several interpolation techniques for base expected loss. Numerical results are given in section 7. Finally our conclusions are presented in section 8.

# 2 Generic 1-Factor Model

The so called 1-factor Gaussian copula model is in widespread use. In what follows we give a brief description of the algorithm. We refer to Andersen and Sidenius [AS05] for an excellent survey on the subject.

## 2.1 CDO Tranche Valuation

Consider a portfolio of $N$ firms and fix a time horizon $T$. For any $0 \leq t \leq T$ the default times $\tau_i$ and default intensities $\lambda_i(t)$, $i = 1, \ldots, N$, satisfy

$$\mathbb{P}(\tau_i > t) = \exp\left(-\int_0^t \lambda_i(u)\mathrm{d}u\right) \tag{1}$$

where $\mathbb{P}$ is the risk-neutral probability measure. In a 1-factor model of portfolio defaults, a single systemic factor $X$ is introduced, conditional upon which all default probabilities are independent.

[2]Base or equity tranches are tranches with attachment point 0.

The single name survival probabilities $\mathbb{P}(\tau_i > t)$ are typically implied from the credit default swap (CDS) market. The fair spread of a CDS balances the present value of the contingent leg $C$, given by

$$C = N^{\text{CDS}}(1-R)\sum_{i=1}^{n} d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right) \tag{2}$$

and the present values of the fee $F$ leg, given by

$$F = N^{\text{CDS}} S\left(\sum_{i=1}^{n} P_S(t_i) d(t_i)\Delta t_i + A_D\right), \tag{3}$$

where $N^{\text{CDS}}$ is the CDS notional and $A_D$ is the accrual on default

$$A_D = \frac{1}{2}\sum_{i=1}^{n} N^{CDS} d(t_i)\left(P_S(t_{i-1}) - P_S(t_i)\right)\Delta t_i. \tag{4}$$

In these equations the summations run over the payment dates, $S$ is the spread premium on a yearly basis, $P_S(t_i)$ is the survival probability at time $t_i$, $R$ is the recovery rate, $d(t)$ is the risk-free discount factor and $\Delta t_i = t_i - t_{i-1}$ is the year fraction.

The key step in valuing CDO tranches is to compute the joint loss distribution. We follow Andersen et al. [ASB03] and compute by means of a simple recursion formula a discretised version of the conditional loss distribution. A loss unit $u$ is chosen so that within a certain tolerance, losses can be represented by integers. For the iTraxx Europe Main portfolio with an assumed uniform recovery rate of 40%, the loss unit is $0.48\%$. We denote by $P^{(i)}(l,t|X)$ the probability of $l$ losses (in terms of the loss unit $u$) at time $t$ with $i$ names conditional on the market factor $X$. Recalling that conditional on $X$ all default probabilities are independent, we can write that $P^{(i)}(l,t|X)$ is the sum of two terms:

$$P^{(i)}(l,t|X) = P^{(i-1)}(l,t|X)\mathbb{P}(\tau_i > t|X) + P^{(i-1)}(l-\omega_{(i)},t|X)(1-\mathbb{P}(\tau_i > t|X)), \tag{5}$$

where $\omega_{(i)}$ is the loss in terms of the loss unit $u$ incurred by a default of the $i$th name. The unconditional loss distribution is found by integrating over the market factor

$$P(l,t) = \int_{\Omega_X} P(l,t|X) f(X)\mathrm{d}X, \tag{6}$$

where $f(X)$ is the density of the probability distribution of the market factor $X$. The fair spread of a CDO tranche balances the present value of the fee $F$ leg, given by

$$F = S\sum_{i=1}^{n}(N^{(\text{Tr})} - \mathbb{E}[L_i^{(\text{Tr})}])d(t_i)\Delta t_i \tag{7}$$

and the present value of the contingent leg $C$, given by

$$C = \sum_{i=1}^{n} d(t_{i+\frac{1}{2}})\left(\mathbb{E}[L_i^{(\text{Tr})}] - \mathbb{E}[L_{i-1}^{(\text{Tr})}]\right). \tag{8}$$

In these equations the summations run over the payment dates, $S$ is the spread premium on a yearly basis, $d(t)$ is the risk-free discount factor, $\Delta t_i = t_i - t_{i-1}$ is the year fraction, $\mathbb{E}[L_i^{(\mathrm{Tr})}]$ is the expected loss on the tranche at time $t_i$ and $N^{(\mathrm{Tr})}$ is the tranche size. For the [0–3%] equity tranche the spread is fixed at 500 basis points[3] and the upfront value is quoted.

In the base correlation framework, the expected loss on a tranche is computed as the difference of the expected loss of two equity tranches

$$\mathbb{E}L[\text{A–D}] = \mathbb{E}L[\text{0–D}; \rho_D] - \mathbb{E}L[\text{0–A}; \rho_A].$$

For more details on base correlation we refer to McGinty et al. [MBAW04] and to O'Kane and Livasey [OL04].

Other approaches to compute the joint loss distribution exist. We refer to Moody's [DSFT03] for models based on Fast Fourier Transform (FFT) and to Yang et al. [YHZ06] for models based on the Saddle Point Approximation.

## 2.2 Gaussian Copula

In the 1-factor Gaussian copula model, default occurs if the latent variable

$$A_i = \sqrt{\rho}X + \sqrt{1-\rho}X^{(i)}, \quad i = 1, \ldots, N \tag{9}$$

falls below some threshold $K_i$. These threshold $K_i$ are constructed so that market CDS prices are replicated. The market or systemic factor $X$ and the idiosyncratic factor $X^{(i)}$ are taken to be independent standard normal random variates. Gauss-Hermite quadrature can be used to evaluate the integral in (6).

## 2.3 Generic One-Factor Lévy Model

This section is a short summary of a few sections in our earlier paper [GGMS07]. For more details on Lévy processes we refer to Bertoin [Ber96] and Sato [Sat00]. We limit ourselves to the following definitions to establish notation.

If for every positive integer $n$, the characteristic function $\phi(u) = \mathbb{E}[\mathrm{i}uX]$ is also the $n$th power of a characteristic function, the distribution is said to be infinitely divisible. Given an infinitely divisible distribution, a stochastic process, $X = \{X_t, t \geq 0\}$, can be constructed. This so-called Lévy process starts at zero $X_0 = 0$, has independent and stationary increments and the distribution of the increments $X_{t+s} - X_s$, has $(\phi(u))^t$ as characteristic function.

In the present setting we work with Lévy processes only on the unit time interval. The cumulative distribution function of $X_t$ and its inverse are denoted by $H_t$ and $H_t^{[-1]}$ respectively. We normalise the distribution so that $\mathbb{E}[X_1] = 0$ and $\mathrm{Var}[X_1] = 1$. Hence one has $\mathrm{Var}[X_t] = t$. We take $X$ and $X^{(i)}$, $i = 1, 2, \ldots, N$ independent and identically distributed Lévy processes.

In the generic 1-factor Lévy model, default occurs if the latent variable

$$A_i = X_\rho + X_{1-\rho}^{(i)}, \quad i = 1, \ldots, N \tag{10}$$

[3] A basis point is equal to 0.01%.

falls below some threshold $K_i$. Each $A_i$ has the same distribution function $H_1$. Note that for $i \neq j$, we have $\text{Corr}[A_i, A_j] = \rho$. In order to match default probabilities under this model with default probabilities $p_i(t)$ implied from the CDS market, we set the threshold

$$K_i(t) = H_1^{[-1]}(p_i(t)).$$

Denote by $p_i(y;t)$ the conditional default probability of firm $i$ given the value $y$ for the systemic factor. From (10) we have

$$p_i(y;t) = H_{1-\rho}(K_i(t) - y).$$

Note that the Gaussian copula model is a special case of the generic 1-factor Lévy model, in which the normal distribution is used.

### 2.4 The Shifted Gamma Model

The Gamma$(a, b)$ distribution with parameters $a, b > 0$ is infinitely divisible. Its density function is given by

$$f(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-xb), \tag{11}$$

for $x > 0$. The Gamma-process $G_t$ is defined as the stochastic process which starts at zero and has stationary, independent Gamma-distributed increments. The time enters in the first parameter, that is $G_t$ follows a Gamma$(at, b)$ distribution. We set

$$X_t = \sqrt{a}t - G_t. \tag{12}$$

It is clear that $X_t$ normalised in the sense that $\mathbb{E}[X_1] = 0$ and $\text{Var}[X_1] = 1$ since Gamma distributions with $b = \sqrt{a}$ have variance 1.

Both the cumulative distribution function $H_t(x; a)$ of $X_t$, and its inverse $H_t^{[-1]}(y; a)$, can easily be obtained from the Gamma cumulative distribution function and its inverse. Gauss-Laguerre quadrature can be used to evaluate the integral in (6).

## 3 Base Correlation Interpolation

The easiest method to set up is piece-wise linear interpolation between $3\%$ and $22\%$. Extrapolation outside this interval can result in negative values or values larger than 1. Applying a floor at 0 and cap at 1, introduces sudden and troublesome changes in the derivative of the base correlation curve. Prescribing values at $0\%$ or $100\%$ is just an arbitrary choice. It is well known that a sudden change in the derivative of the base correlation curve can result in a sudden change in the fair spread of tranchlets and even worse in model arbitrage. In order to avoid this model arbitrage problem, interpolation methods that produce continuous derivatives are preferred. The most notable method being cubic spline interpolation, which produces continuous curves having continuous first and second derivatives. However the extrapolation issues outside the $3\%$-$22\%$ region remain. Even smooth base correlation curves may be arbitragable and it is impossible to know it unless all tranchlet prices are checked. It is clear that interpolating base correlation curves has some disadvantages.

# 4 Lévy Base Correlation

The shifted gamma model outlined in section 2.4 has 2 parameters. The parameter $a$ in (12) and the correlation parameter $\rho$ in (10). A global model calibrates these two parameters to the 5 observed tranche spreads. It is clear that sensitivity to $a$ and $\rho$ is quite different. Hence both $a$ and $\rho$ can be used to explain spread movements, e.g. the May 2005 auto crisis.

The global model with 2 parameters $a$ and $\rho$ is unable to match the 5 observed tranche spreads exactly. As in the Gaussian copula case, we move to the base correlation methodology and we use a base correlation value for every (base) tranche. For iTraxx we have 5 base correlation values $\rho_3$, $\rho_6$, $\rho_9$, $\rho_{12}$ and $\rho_{22}$. Now we are left to calibrate 6 parameters to the 5 observed tranche spreads. Several calibration strategies are possible. We could impose an additional constraint, such as requiring that the base correlation be equal for the equity and the junior mezzanine tranches ($\rho_3 = \rho_6$), or we could set an additional optimisation target, such as smoothness of the base correlation curve and minimise slope of the base correlation curve. All these strategies have the same annoyance: one parameter is of different type. The $a$ parameter is not a (base) correlation value, but rather characterises the Gamma distribution used.

To overcome this we set $a = 1$ and proceed along the base correlation methodology. Fixing $a = 1$ results in the Exponential(1) distribution being used for $G_1$. We motivate this choice as follows. First of all we present a distributional argument. Recall that the normal distribution has a density proportional to $\exp(-0.5x^2)$ and consequently decays very fast. The exponential distribution has a density $f(x) = \exp(-x)$ for $x \geq 0$ and it is the typical representant of the next class of fatter (than normal) tail distributions. It turns out that moving into this class of distributions is a key building block. Second $a = 1$ is a typical value in free $a$ calibration. Finally it is the most tractable choice.

In what follows Lévy base correlation is defined as the base correlation in the shifted gamma model with fixed $a = 1$. For more details on Lévy base correlation and a comparison with the Gaussian base correlation we refer to our earlier paper [GGMS07] and the references therein. Another example of a Lévy based model is given by Hooda [Hoo06]. He has shown that the Normal Gamma copula base correlation curve is less skewed than the Gaussian copula base correlation curve.

# 5 Base Expected Loss

We define base expected loss $l(x)$ as the expected loss of an equity tranche at maturity

$$l(x) = \mathbb{E}[\text{Loss}(0, x, \rho_x, T)]. \tag{13}$$

This quantity is readily available in the base correlation framework. Note that this definition is different from what Parcell and Wood [PW07] define as discounted base expected loss, which is the present value of the contingent leg in (8). Base expected loss $l(x)$ is non-decreasing as a function of the attachment point $x$, since its derivative corresponds to a probability:

$$\frac{\partial}{\partial x} l(x) = \mathbb{P}[\text{Loss} \geq x] \geq 0. \tag{14}$$

Base expected loss $l(x)$ cannot be positively convex, since its second derivative is minus the density of the loss distribution:

$$\frac{\partial^2}{\partial x^2} l(x) = -f_{\text{Loss}}(x) \leq 0. \tag{15}$$

Similar properties on expected loss related quantities have also been described by Torresetti et al. [TBP07] and by Parcell and Wood [PW07].

An advantage of a loss based approach is that boundary values are available. Clearly we have that $l(0) = 0$ and $l(1) = \mathbb{E}[\text{Loss}(0, 100\%, T)]$ is the expected loss on the underlying pool, independent of correlation, determined only by spreads and recovery rates. As a side product of bootstrapping implied base correlations points, we also get the base expected losses for the standard attachment points. Starting from these values and the properties (14) and (15), we can construct an upper and a lower bound for base expected loss interpolations.

The lower bound is the curve, obtained by piece-wise linear interpolation on the values already obtained. This can easily be seen by a convexity argument. Several upper bounds can be given. Clearly base expected loss is bounded by the size of the tranche $l(x) \leq x$ and by the expected loss on the complete pool of underlying obligors $l(x) \leq l(1)$. Suppose $0 \leq p < q < r \leq 1$ are neighbours in the sequence of standard attachment points, that is there is no standard attachment point between $p$ and $q$ and between $q$ and $r$. In this case the straight line through the points $(p, l(p))$ and $(q, l(q))$ is an upper bound on $l(x)$ for $x \geq q$ and the straight line through the points $(q, l(q))$ and $(r, l(r))$ is an upper bound on $l(x)$ for $x \leq q$. This can also be seen by convexity arguments. The upper bound is of course the minimum of all applicable upper bounds. Note that the lower bound satisfies (15) but the upper bound does not, it is the envelope of a set of functions that do. Thus the upper bound is not a feasible base expected loss curve.

# 6 Base Expected Loss Interpolation

As mentioned above piece-wise linear interpolation on the base expected losses is the lower bound for any interpolated base expected loss curve. It is an acceptable method in the sense that it satisfies all the constraints. However we want a smooth base expected loss curve with a continuous derivative, since the derivative of the base expected loss curve is related to the probability distribution of the losses. The convexity constraint (15) prescribes that this derivative be non-increasing.

We want a smooth interpolated base expected loss curve so cubic spline interpolation is certainly a candidate to be considered. Unfortunately a cubic spline interpolant can exhibit oscillations, thus violating the monotonicity constraint (14). However this does not rule out smooth cubic interpolation for the base expected loss curve. The idea is to give up on the constraint imposing continuity of the second derivative and impose other constraints, related to monotonicity and convexity.

Our first interpolation approach is as outlined by Fritsch and Carlson [FC80]. On each subinterval a cubic interpolant is used and continuity of the first derivative is imposed. The slopes at the data points are chosen to preserve the shape of the data and to respect monotonicity. The second derivative is not necessarily continuous, as it may jump at the control points.

Our second interpolation scheme is to use cubic spline and apply the monotonicity filter as described by Dougherty et al. [DEH89] to it. This approach has the advantage of leaving the interpolating spline unchanged when it is monotonic, resulting in a very smooth interpolant.

Another candidate is the monotonic interpolation introduced by Steffen [Ste90]. Contrary to spline interpolation this method has the advantage of being local. Since a third order polynomial is being used and monotonicity is enforced, convexity cannot be guaranteed. Parcell and Wood [PW07] also present a piece-wise quadratic interpolation method. They enforce the monotonicity and convexity constraints. Note that piece-wise quadratic interpolation for base expected loss translates into piece-wise constant interpolation for the density of the loss distribution, which is a rather crude approximation.

# 7 Numerical Results

In this section we present numerical results for iTraxx Europe Main. The market data is taken on June 22, 2007. We look at the 10 year contract.

In Fig. 1 we show the base expected loss curves for the Gaussian copula. We calibrate to the observed market tranche spreads and we get the base correlations and base expected loss values for the standard attachment points. For the base expected loss we also have the boundary values at 0 and at 100%. Starting from these values and the properties (14) and (15), we construct the upper and the lower bound for base expected loss interpolations as described in section 5. The red lines show these bounds. It is clear that these bounds interpolate in the bootstrapped values. We also show the cubic spline interpolation with application of the monotonicity filter as described in section 6. This is the blue line. Finally, the green line shows the shape preserving cubic interpolation. Note that for small values of the attachment point, this interpolation violates the upper bound. Both interpolations satisfy the convexity constraint (15).

Fig. 2 shows the base expected loss curves for the Lévy model. Note that also here for small values of the attachment point, the shape preserving cubic interpolation violates the upper bound. Both interpolations satisfy the convexity constraint (15). Comparing this figure to Fig. 1, we see some differences between the Lévy model and the Gaussian copula model. For the 0–3% tranche the Gaussian copula model gives a base expected loss at maturity of 0.0252 (84%), while the Lévy model gives a base expected loss at maturity of 0.0261 (87%). The expected loss on the complete pool is 0.0475, independent of the model used to price the tranches. Both the Lévy and the Gaussian copula models are calibrated to the same observed tranche spreads. Hence the difference between the models is clearly in the distribution of the expected losses over the tranches.

The Gaussian copula base correlation curves are shown in Fig. 3. First of all we have base correlations values for the standard attachment points as a result of the calibration to the observed market tranche spreads. We are not interpolating base correlation directly. As explained above we have bounds on base expected loss and we interpolate the base expected loss curve. For a given attachment point and corresponding base expected loss value, we calibrate for the corre-

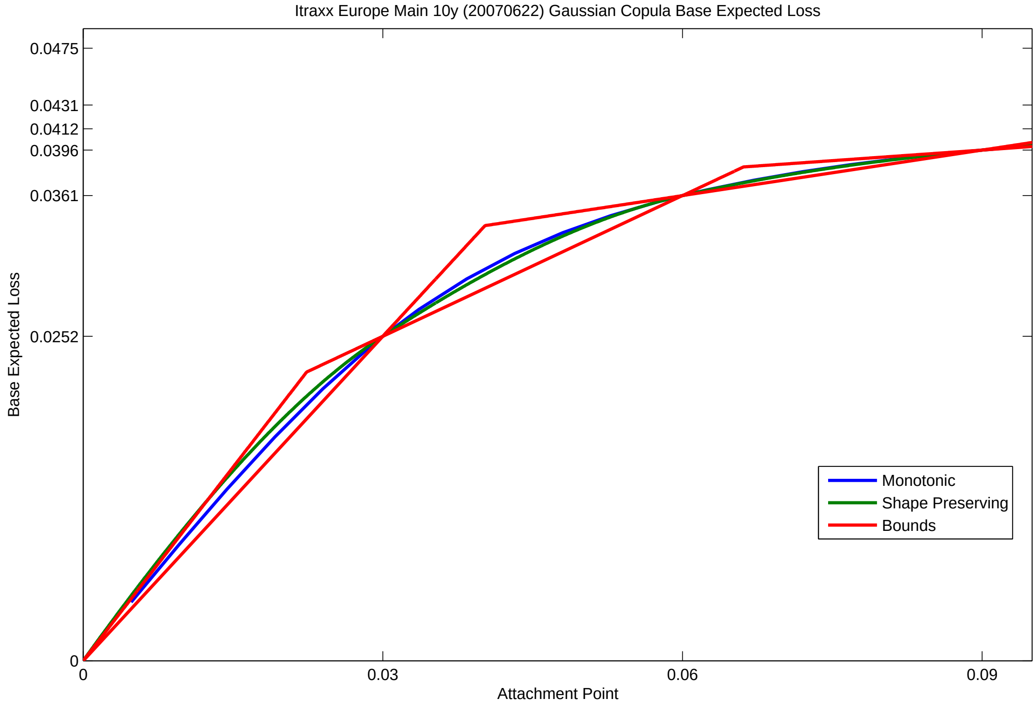


Figure 1: Gaussian Copula Base Expected Loss. The red lines show the upper and lower bounds. The blue line shows the monotonic cubic spline interpolation and the green line shows the shape preserving cubic interpolation.

sponding base correlation value. As there is a one-to-one relationship between base expected loss and base correlation, the bounds on the former imply bounds on the latter. These are shown as the red lines. Recall that the upper bound on base expected loss is not a feasible base expected loss curve. Consequently the implied base correlation curve is probably not arbitrage-free. The calibration for base correlation is also done when shape preserving cubic interpolation is done on the base expected loss curve. This results in the green line. Observe that the base correlation curve hits zero in this case. Finally, the blue line shows the implied base correlation curve when cubic spline interpolation with application of the monotonicity filter is done on the base expected loss curve. This results in a base correlation smile.

Fig. 4 shows the Lévy base correlation curves. Also here the base correlation hits zero when shape preserving cubic interpolation is done on the base expected loss curve. Comparing this figure to Fig. 3, we see some differences between the Lévy model and the Gaussian copula model. Lévy base correlation is very flat above 3% and can become quite large for very small attachment points. Gaussian copula base correlation is much steeper above 3% and also smiles below 3%, but less than Lévy base correlation.

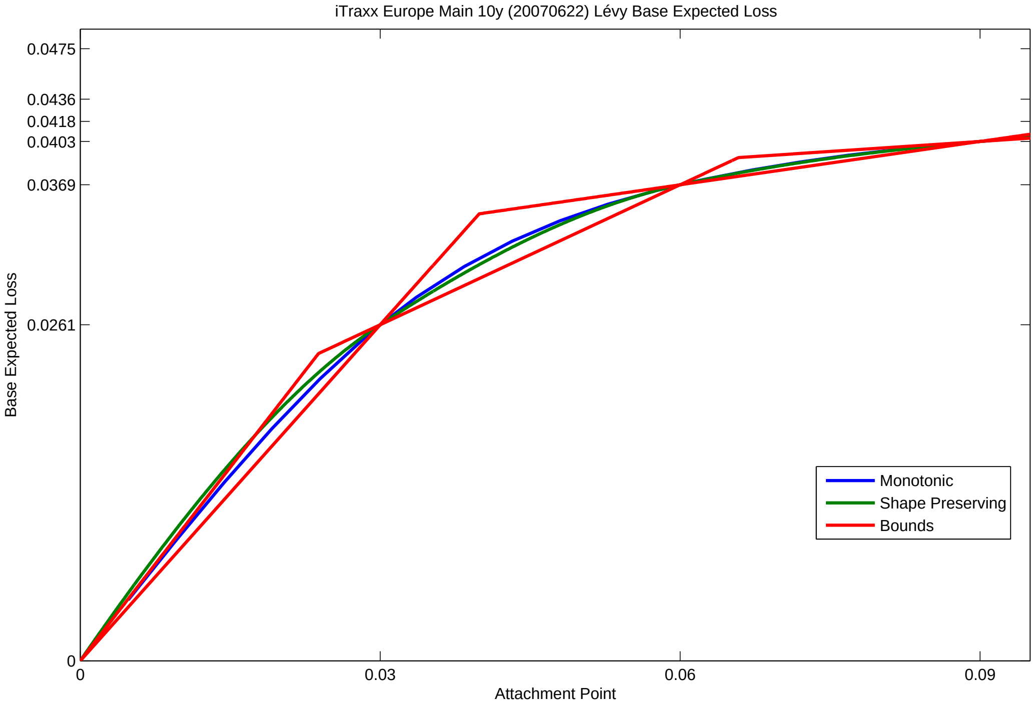


Figure 2: Lévy Base Expected Loss. The red lines show the upper and lower bounds. The blue line shows the monotonic cubic spline interpolation and the green line shows the shape preserving cubic interpolation.

# 8 Conclusions

In this paper we have introduced base expected loss and stated the monotonicity and convexity constraints it needs to obey. Based on these constraints we derived upper and lower bounds an interpolation scheme for base expected loss needs to satisfy in order to avoid model arbitrage. We studied the effect of interpolating base expected loss curves both in the Gaussian copula and in the Lévy 1-factor models for CDO tranche valuation. As there is a one-to-one relationship between base expected loss and base correlation, we also show the implied base correlation curves. The best results have been obtained using a cubic spline interpolation with a monotonicity filter applied to it on the base expected loss curves. The implied base correlation curves show a typical smile, both for the Gaussian copula and the Lévy 1-factor models. In the Gaussian case the base correlation smile is less pronounced in the [0–3%] range and is more prounounced in the senior tranches. In the Lévy model on the other hand the base correlation smile is more pronounced in the [0–3%] range and is less prounounced in the senior tranches. Additionally a very important result of this study is a practical algorithm to evaluate spreads for tranchlets outside the [3%–22%] range, since this is a well-behaved interpolation in the base expected loss framework

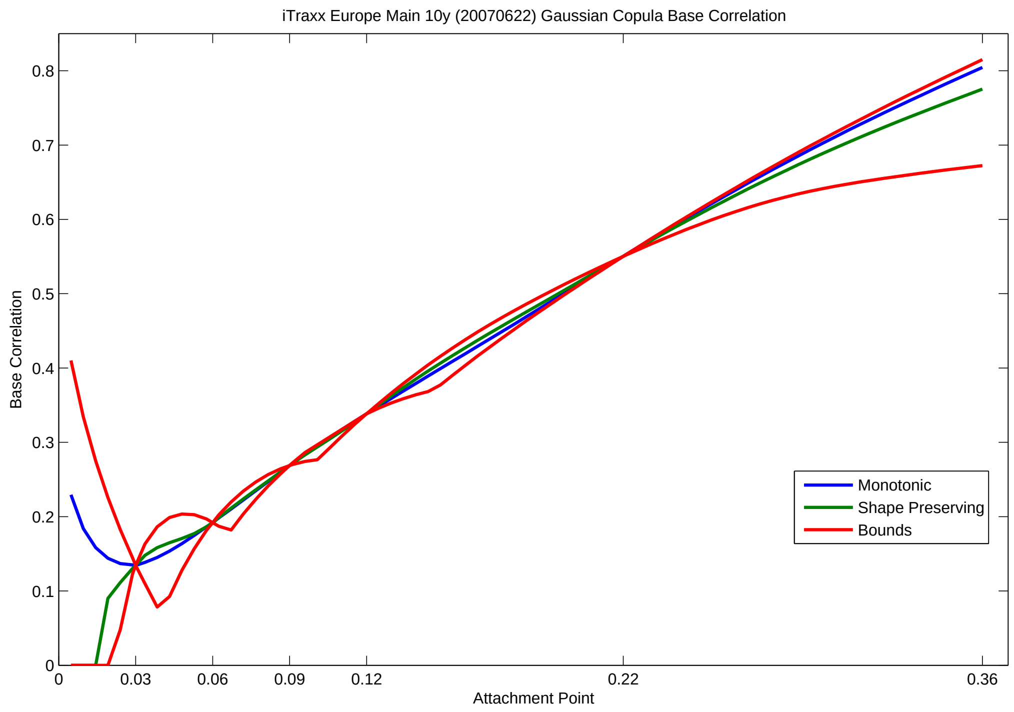


Figure 3: Gaussian Copula Base Correlation. The red lines show curves implied from the upper and lower bounds on base expected loss. The blue line shows the curve for monotonic cubic spline interpolation and the green line shows the shape preserving cubic interpolation.

instead of a dangerous extrapolation in the base correlation framework.

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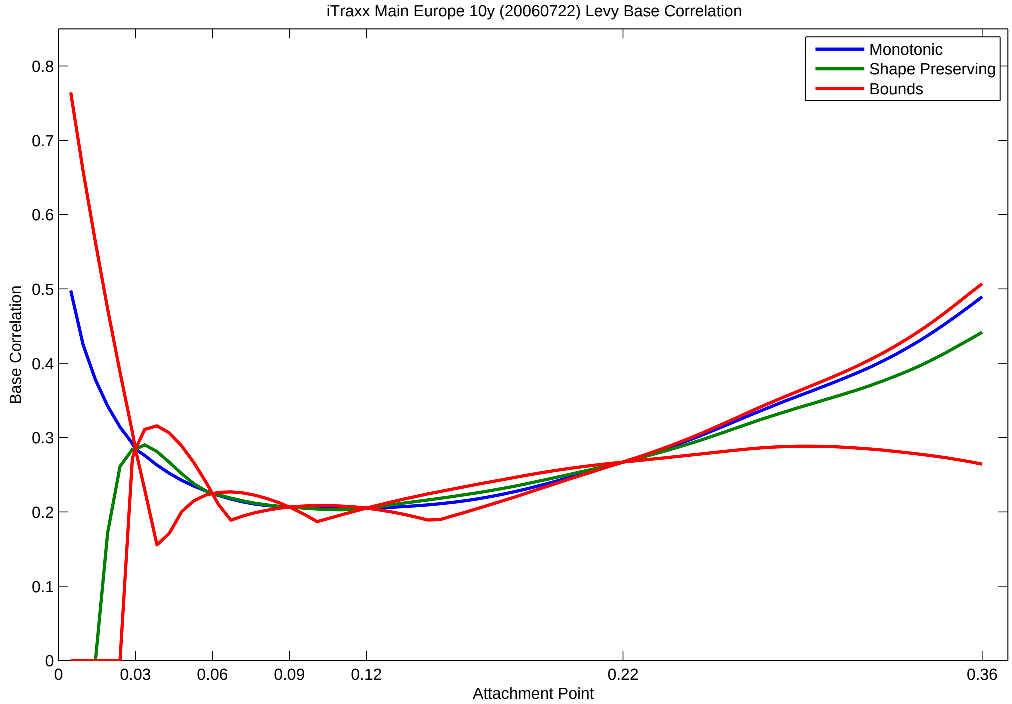


Figure 4: Lévy Base Correlation. The red lines show curves implied from the upper and lower bounds on base expected loss. The blue line shows the curve for monotonic cubic spline interpolation and the green line shows the shape preserving cubic interpolation.

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